German-Greek Yearbook of Political Economy, vol. 2 (2018)

ARS COMBINATORIA v2.5 or: The 64 Ways of Order

Konstantinos Koulaouzidis

Munich based artist, www.koulaouzidis.com
koulaouzidis@gmail.com

Abstract: *The driving force behind this project is the search for the possibility to relate and understand seemingly different or dissimilar ideas or disciplines by tracing common patterns and analogies. It is the Pythagoreans who were also open to the idea of achieving this goal with "patterns" as a possible means: a basic concept on which, according to artists and scientists, not only the sciences are based, but human society and, in the broadest sense, life and the whole cosmic order. Patterns and numbers are inextricably linked together. 64 very special magic squares are the foundation of the project. They are used in order to correlate numbers, colors, sounds, the binary system (in form of the I Ching) and the base triplets of the essential amino acids. The long-lasting research and combinatorial studies aim to open up dialogues between the realms of science and art, as well as to give a personal artistic interpretation of the aforementioned correlations by means of different forms of presentation.*

Keywords: science art, math art, numbers, colors, sounds, magic squares, binary system, I Ching, genetic code, creative coding, generative art, generative music, symmetry, pattern

Color version online:
www.koulaouzidis.com/ARS-COMBINATORIA_v2.5_K.Koulaouzidis.pdf
www.CCR-Munich.de

Updated and future versions online:
www.koulaouzidis.com/ARS-COMBINATORIA_K.Koulaouzidis.pdf

ISBN 978-3-88278-301-8
www.accedoverlag.de

1. Introduction

"He who understands the numbers (that is: their innermost essence), understands everything."

Plato (Timaeus)

The Pythagoreans were convinced that the principles of numbers are the principles of all things in this world. They claimed that "the cosmos is isomorphic with pure mathematics" (Bell)[1] and that everything in the universe can be measured by whole numbers.

For Augustine, numbers also occupy a very special place in the world order. According to him, the numbers were already there before the world came into existence. The abstraction inherent in the figures makes it possible to represent or express almost everything with a clarity that would not be possible with words.

Artists and mathematicians alike have long recognized that numbers play a very important role in art (i.e., golden ratio in design and architecture, number ratios in music scales). On that account, the Italian mathematician Luca Pacioli explained 500 years ago: "Without mathematics, there is no art."

At least since Pythagoras there is the belief in a mathematically ordered, harmonious world, a universal theory that explains and connects everything. The vision of a Harmonia Mundi or a universal harmony of the spheres again and again derived from proportional correspondences and was transferred into harmonical systems. In the twentieth century, this was especially the result of Hans Kayser's studies. The belief in a mathematically ordered world has even led Johannes Kepler to some of his discoveries: He had the "unshakably self-evident conviction of a harmony between man, earth, and space ruled by the number – the certainty of a mystical significance of the number" (Hartner).[2] The Italian Renaissance artist Francesco Giorgio used numerology as some kind of super-discipline in his Harmonia Mundi (1525), with the help of which all other disciplines were to be united.

The driving force behind my project is the search for a possibility to relate and understand seemingly different or dissimilar ideas or disciplines by tracing common patterns and analogies. One should also bring to mind

[1] Endres and Schimmel (2001: 24).
[2] Ibid. (2001): 38.

Hermann Hesse's "Glass Bead Game" in which the technique of the Glass Bead Game, developed and practiced in fictional Castile, aims at an interdisciplinary integration of art and science. Mathematics and music, and the humanities and natural sciences, drifting apart by advancing specialization, can be reconnected by using the universal sign language of the Glass Bead Game, and can become aware of their common structure.

The Pythagoreans were also open to the idea of achieving this goal by using "patterns" as a possible means: a basic concept on which, according to artists and scientists, not only the sciences are based, but also human society and, in the broadest sense, life and the whole order of the universe.

According to Andreas Goppold (2017) "the term pattern has gained prominence as key term in understanding mankind's quest to make the universe intelligible, to fashion a Cosmos from the pure Chaos of the indiscriminate swarm of photons, electrons, air pressure changes, chemical and physical stimulants, that organisms are exposed to every instant of their living existence."

Patterns and numbers are inextricably linked together. This "in-evitability" of patterns in the universe[3] suggests that there must be descriptions for all of them. In order to find them, we need a catalog or a list of all possible patterns. This catalog is called mathematics.

2. Project Background

"Everything is ordered by measure, number and weight,
and the laws are without change."

Old Testament (Wisdom 11:20)

I have always been interested in and fascinated by numbers, colors, sounds and patterns. At a young age, I was already looking for organizing structures that could relate everything with everything. Many scientists and artists have practiced that before me – however, their attempts were often based on intuition, esoterism, etc.

As a scientifically minded person, I wanted to create my own logical system based on existing principles. My idea was that if I managed to detect a system that perfectly and harmoniously arranged elements, I could project colors and sounds on it. And this arrangement of colors and sounds would then likewise be perfect and harmonious.

[3] Barrow (1998: 192).

Some time ago, numbers with interesting, even fascinating features attracted my attention – the so-called "Magic Squares" of the fifth order (5x5), whose numbers exhibit a perfect inherent harmony. I used 64 special squares as the foundation of the project in order to correlate colors with sounds. Furthermore, I discovered other relevant principles. The number 64, the project's central number, occurs both in the I Ching (64 hexagrams), as well as in the genetic code (64 base triplets of the essential amino acids). This fact led me to the work of Martin Schönberger (2000) who connects both. The hexagram structure of the I Ching and its connection to the binary system confirmed my (originally intuitively) derived foundation of the project. I thereupon included the binary system, the I Ching and the 64 base triplets of the essential amino acids in my project. This will be described more detailed in the following sections.

3. Detailed Project Description

"Numbers therefore are endowed with great and sublime virtues. For it is no wonder, seeing there are so many, and so great virtues in natural things, although of manifest operations, that there should be in numbers much greater, [...] for as much as they are more formal, more perfect, and naturally in the celestials. [...] Again, all things that are, and are made, subsist by, and receive their virtue from numbers.For time consists of number, and all motion, and action, and all things which are subject to time, and motion."

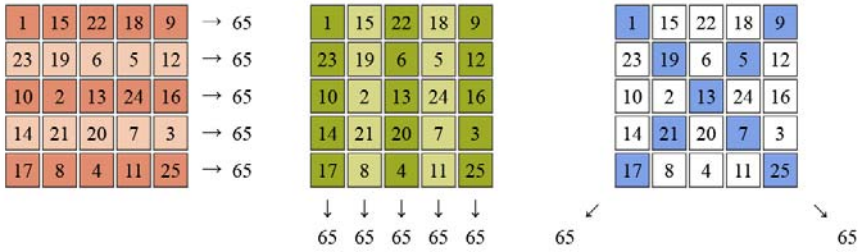
Agrippa of Nettesheim (1486-1535), "De occulta Philosophia"

3.1 Magic Squares: An Introduction#

#

A magic square is a n x n square grid (where n is the number of cells on each side) filled with distinct positive integers in the range 1, 2, 3, ..., n^2 such that each cell contains a different integer and the sum of the integers in each row, column and diagonal, is equal. The sum is called the "magic constant" or "magic sum" of the magic square and is calculated by the formula: $M = (n^3+n) / 2$ The number n is called the "order" of a magic square.

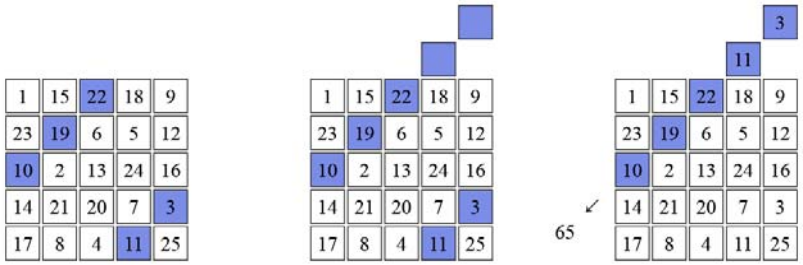
The following visual examples will be based on an order 5 magic square with a "magic constant" of 65.



In addition to the above definition, magic squares can also possess other special features. The ones important for this project are briefly discussed here.

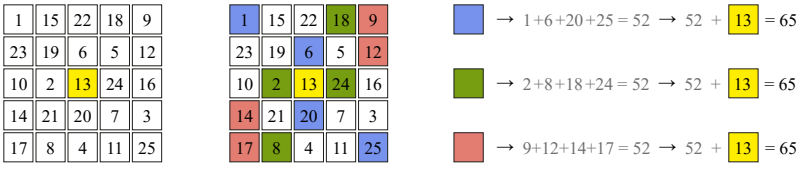
PanMagic (or Pandiagonal Magic) Squares

PanMagic squares have the additional property that the broken diagonals, i.e. the diagonals that wrap around the edges of the square, also add up to the magic sum. They remain pandiagonally magic not only under rotation or reflection, but also if a row or column is moved from one side of the square to the opposite side.



Symmetric (Regular, Associative) Magic Squares of Odd Order

In a symmetric magic square all numbers symmetrically opposite the center sum to $(n^2+1) * 2$. Alternatively, 4 cells symmetrically opposite the center cell plus the center cell add up to the magic sum.



Ultramagic Squares

Magic squares of the 5th order that possess both features mentioned above (panmagic and symmetric) are called "Ultramagic Squares".

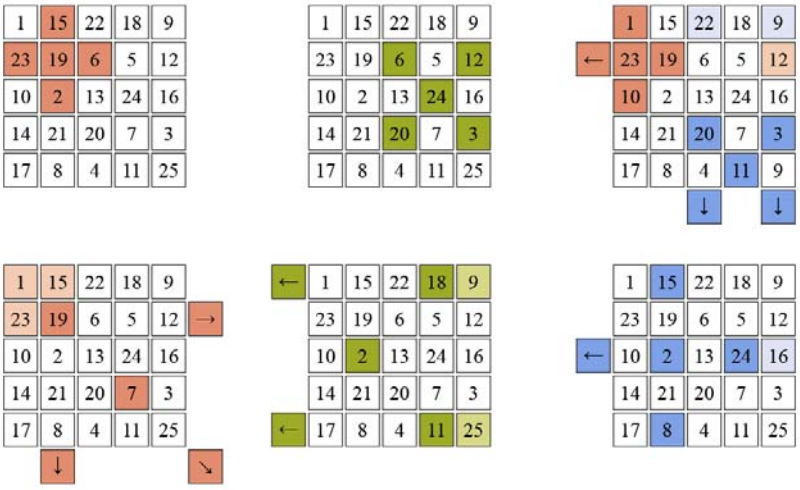
Ultrasupermagic Squares

Code written specifically for this project calculated that there are 1394 combinations of 5 non-repeating numbers, from the first 25 consecutive numbers, that sum to 65. Actually, the 1394 is a subset of the combinations (without repetition) to the k-th class:

$K(n;k) = (n / k) = (25 / 5) = (25! / 20!5!) = 1{,}551121e25 / 2{,}432902e18 * 120 = 53130$
or
$25 * 24 * 23 * 22 * 21 \ / \ 1 * 2 * 3 * 4 * 5 = 53130$

An interesting side effect of the 1394 combinations is that some of them have an additional symmetry feature for some order 5 magic squares: 5 numbers forming an exact cross or x (including wrap-around *) also result in the magic sum, as you can see on the next image.

** Wrap-around means when you go off the square at one edge, continue via-à-vis at the corresponding cell.*



I call magic squares with all the aforementioned features "ultrasupermagic" and they are the best suited foundation for my project. The perfect harmony inherent in the numbers of these squares (the recurrence of the magic sum) becomes visible when all numbers from 1 to 25 are connected by lines. The graphical patterns that arise are always absolutely symmetrical. They can be looked at in the following section.

3.2 The Foundation, or: The 64 Squares of the Project

"…the numbers of the magic square…in this language God had thought before he created the world, and in this language he had said the words around which the world had originated…it remained the memory of the lost unity…"

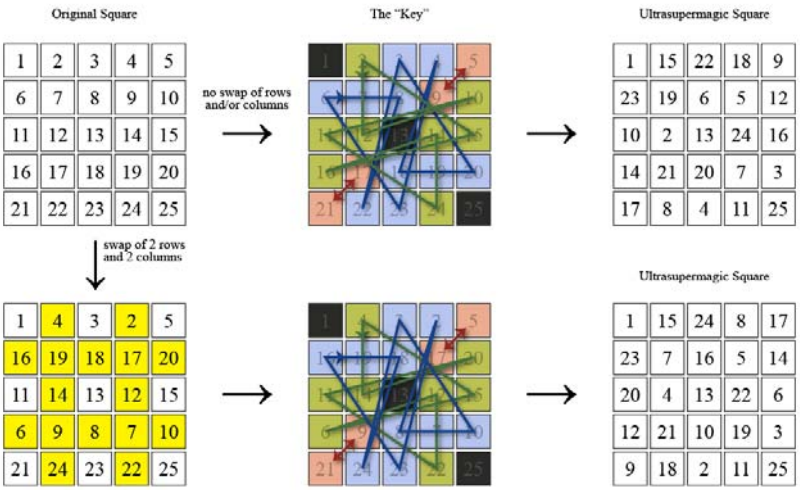
Marc Petit (Uroboros)

As Mitsumi Suzuki calculated with a self-written computer program there are 16 ultrasupermagic squares. His algorithm went through all possible combinations of the numbers 1-25 (a total of over 15 quadrillion is the number 15 with 24 zeros) and found those to which the rules of ultrasupermagic squares applied.

During my work on this project, it became increasingly important to me that every project-related aspect can be derived visually and with simple patterns. Regarding the 16 ultrasupermagic squares to be used as the foundation of the project, I was looking for an elegant, intuitively accessible solution.

Therefore, I was looking for a way to achieve the 16 squares by using visual, logical and combinatorial methods (instead of using a computer program such as Suzuki). Due to the perfect arrangement of the numbers and the symmetrical patterns resulting from their successive connection (see last figure in this chapter), I was sure that there were underlying principles, and I was eager to find them.

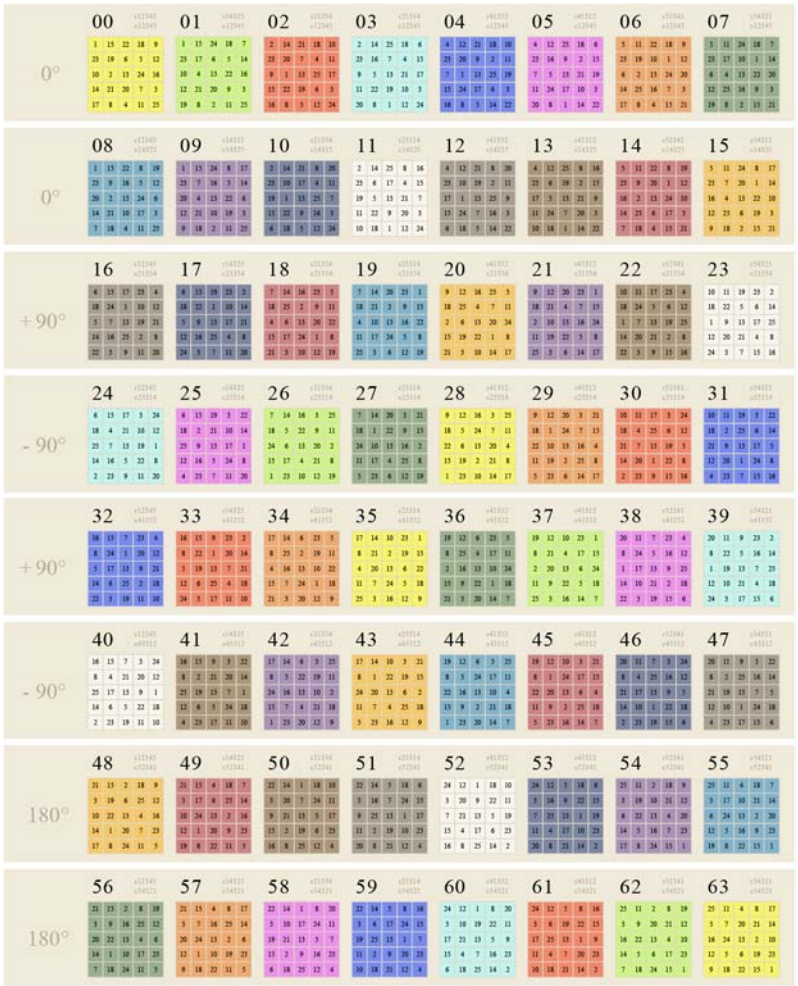
Through reverse engineering (the return of a magic square to its starting square) and other methods I was successful and found out the principles: Certain numbers are never exchanged (marked black in figure below), particular others are moved along a certain pattern (marked in green and blue) and some exchange positions (marked in red). Another important aspect for deriving all squares is that the rows or columns are symmetrically interchanged (swapped) along the respective middle column or row (for the first ultrasupermagic square no rows or columns needed to be swapped).



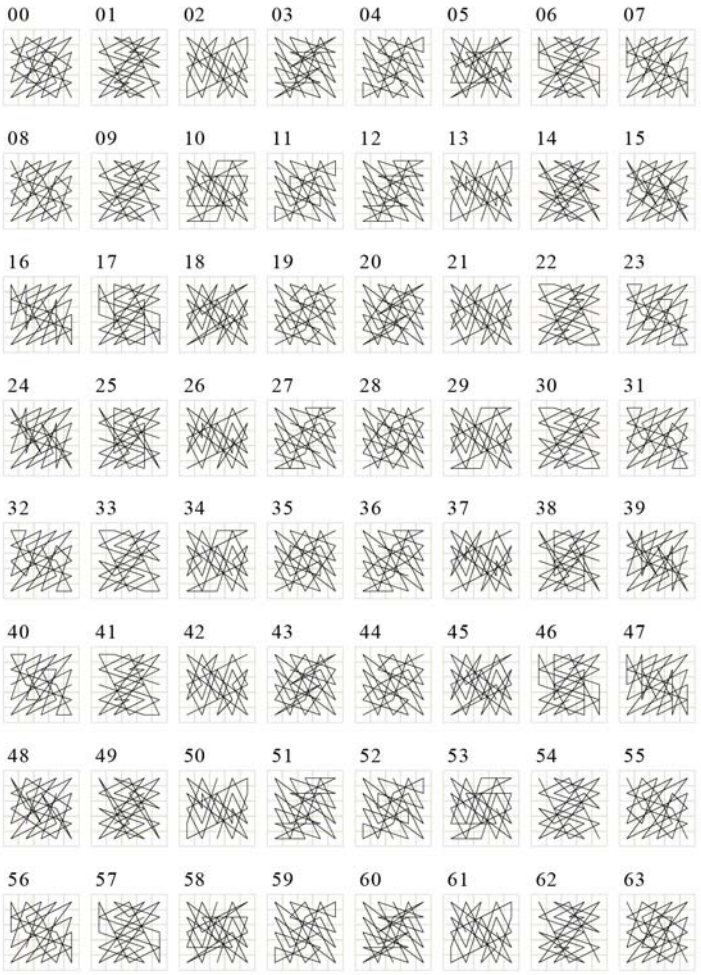
As there are 4 symmetrical swaps (1,2,4,5) we get 8 possible swapping combinations: 1,2,4,5 (no swap) / 1,4,2,5 / 2,1,5,4 / 2,5,1,4 / 4,1,5,2 / 4,5,1,2 / 5,2,4,1 / 5,4,2,1 for the rows or columns, and by multiplying both, we get 64 possible mixing combinations of rows and columns and thus 64 ultrasupermagic squares. This is not contrary to the 16 squares calculated by Suzuki, because if you examine the 64 squares more closely, you realize that 16 squares produce the rest by rotation. Through mirroring along one of the diagonals, their number can again be doubled or tripled. Those newly created squares do not generate any new essential combinations for the project of the numbers 1 to 25 inside the 64 squares (for more information on this see last figure in chapter *Realization and Outlook*).

It should also be mentioned that all possible permutations (without repetition) of the numbers from 1 to 25 give an incredibly large number, that is P(25) = 25! = 15.511.210.043.330.985.984.000.000 or 1.551121e+25 (over 15 quadrillion = the number 15 with 24 zeros), a number that is said to be as large as all the stars in the observable universe. Imagine that only 64 of them possess the inherent perfection and harmony needed for this project!

Below, all squares produced from each other by rotation are colored identically and I call them "rotational siblings". Here you can see the first proof that the chosen swapping sequence of rows/columns was perfect, as it produces a symmetric pattern of the similar squares and as there is an underlying order on the sequence of their appearance on the following 8 rows, sorting them by rotation. The second proof can be found in "3.3 Parallels to the Structure of the Binary System and I Ching".

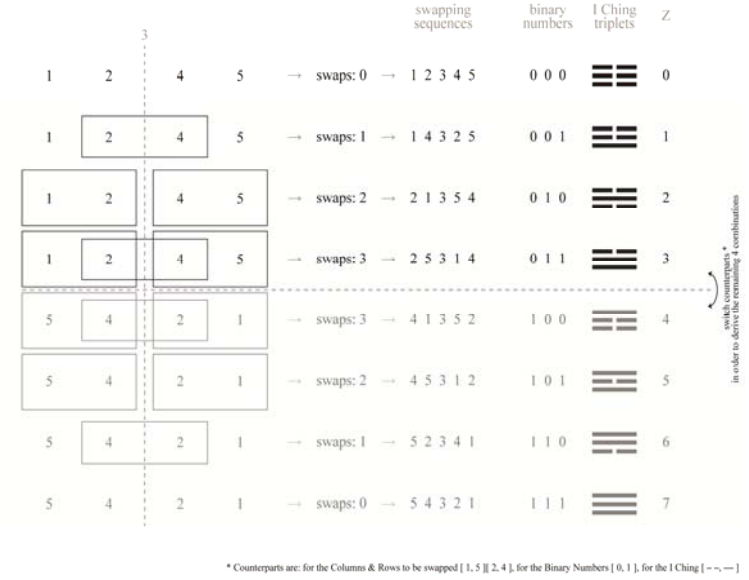


All symmetrical patterns resulting from the successive connection of the numbers 1 to 25 on each of the 64 ultrasupermagic squares. Here again the patterns of the same colored squares are rotations of each other.

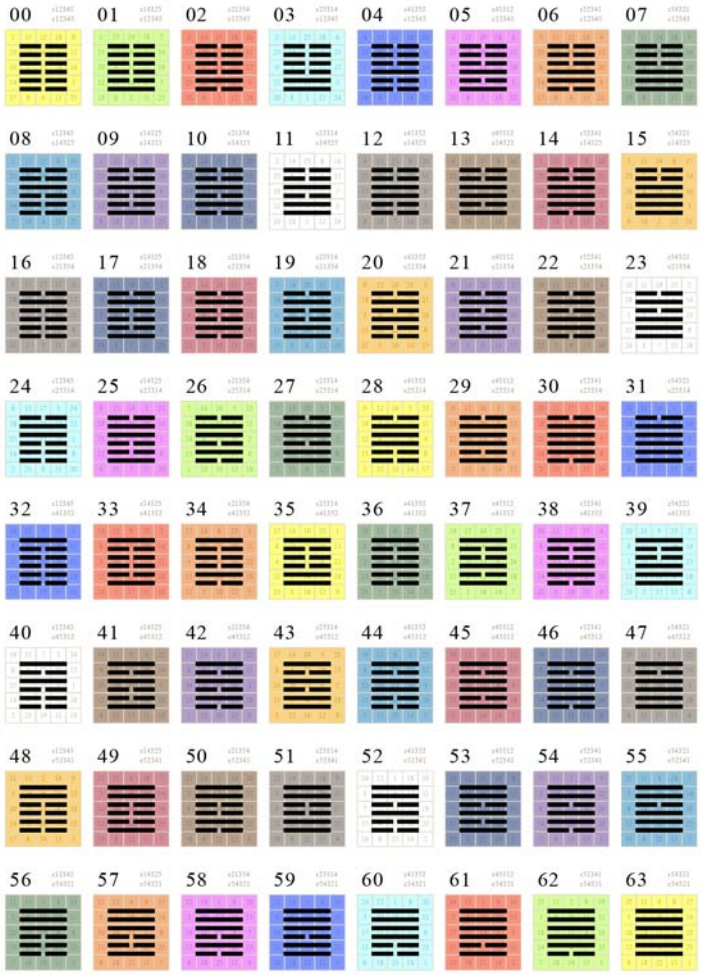


3.3 Parallels to the Structure of the Binary System and I Ching

The number 64 also drew my attention to the I Ching, the Chinese Book of Changes. The I Ching consists of 64 six-part combinations (hexagrams). Even Leibniz, who invented the binary system, had discovered parallels between binary code and I Ching – he comments on this in his "Two Letters on the Binary Number System and the Chinese Philosophy" (Schönberger 2000). I began to use this parallel in order to verify the swapping sequence of the columns and rows (from which the magical squares emerge after applying the "Key", see figure in chapter *Detailed Project Description*). The representation of the I Ching hexagrams or states (which can be understood as binary codes) has helped me to determine the swapping order of the rows and columns – in order to create the ultrasupermagic squares. I discovered that the 64 hexagrams of the I Ching, each consisting of two triplets, can binary-encode the alternations of the columns (1st triplet) and rows (2nd triplet) within a square (a hexagram is being read from bottom to top). See a full list of the 64 ultrasupermagic squares and the corresponding hexagrams on the next page. Again, I was looking for a visual explanation to verify and link the swapping sequence with the I Ching triplets, which I found and tried to visualize on the following figure which shows all possible combinations of 4 symmetrically interchanged (swapped) columns or rows and the parallels of the swapping sequences to the binary system and I Ching.



The 64 ultrasupermagic squares and the corresponding hexagrams of the I Ching.

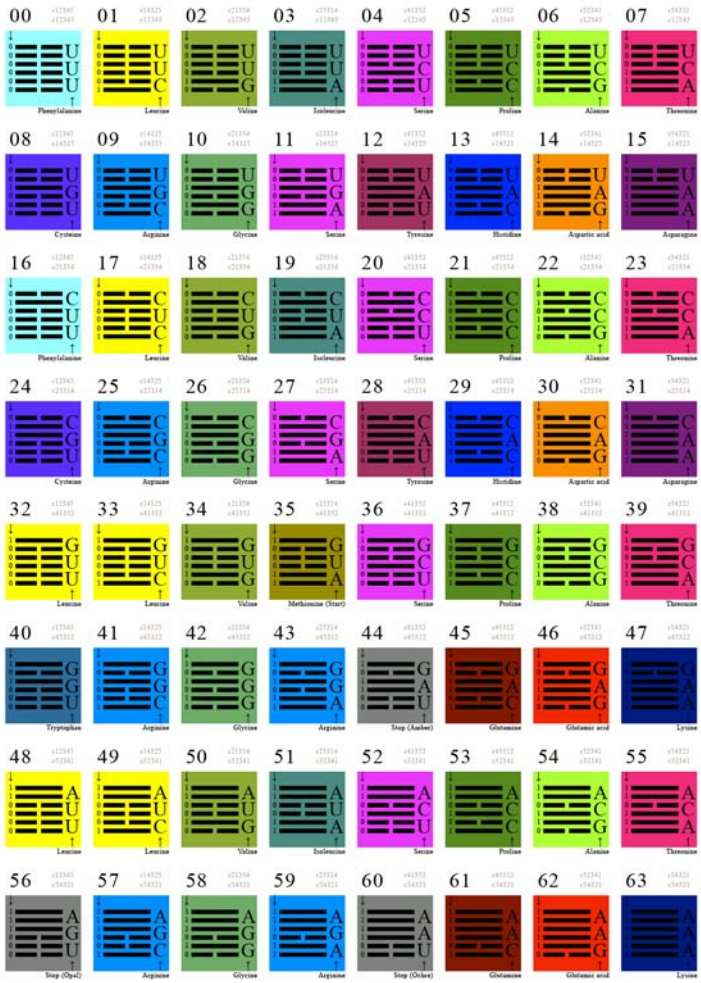


3.4 Parallels to the Structure of the Essential Amino Acids (Genetic Code)

As this is currently only part of the graphic presentation of the project, for now I am only showing a table with the corresponding Square Number > I Ching Hexagram > Binary Code > Nucleotide Triplet (Codon) > Amino Acid. You can see the full list on the next page.

The translation of a codon into a hexagram is based upon the following rule: The Watson-Crick base pairs in RNA molecules (e.g., transfer RNA), guanine-cytosine (G-C) and adenine-uracil (A-U), are mapped to the hexagram as in the following figure. For further details, please consult Schönberger (2000).

For identical amino acids the same color was used. Amino acids having the same biochemical properties (nonpolar / polar, basic / acidic) are colored with similar colors, according to a custom color palette created by taking those properties into account.



3.5 Colors and Sounds

"The primordial nature of color is a dreamlike sound,
Is music turned light." Itten (The Art of Color)

Everything in our world is vibration or energy and can be reduced to waveforms or could even be seen as sound. Or as light or even color. Much has already been written and speculated about the relationship between sounds and colors. Just think of the phenomenon of synesthesia and of the many artists who incorporated the subject in their work (Wagner, Kandinsky, Schönberg, Skrjabin, etc.). Baudelaire, for example, wrote in the "Revue Européenne" (April 1861) about Richard Wagner and Tannhäuser: "…for it would be truly surprising if the tone could not produce the color, if the colors could not produce a melodic motif, and if the tone and the color were not suitable for transmitting thought; especially since things have always been expressed by mutual analogy since the day when God created the world as a complex and indivisible totality" (Maur 1999: 12).

Tables were designed that matched the colors with their respective colors. But none of them was scientifically justified, all based more or less on the personal instincts or perceptions (intuitions) of the persons who created them.[4]

Kandinsky, who was convinced that the phenomenon of "color-listening" actually existed and probably possessed this ability himself, assumed the immediate "touch of the soul". Again and again he was bothered by the analogies of color tones and sound colors, which he tried to assign systematically. Thus, he connected the basic colors with certain instruments (Maur 1999: 30). For him the color is the key, the eye is the hammer and the soul the piano with many strings ("Concerning the Spiritual in Art"). The initial idea was that if you could manage to arrange numbers so that they are perfectly harmonious with each other, you would have the base, even the key, to create a harmonious connection between sounds and colors. I found this key in the "Magical Square" figures explained above. In the next section, the different realizations of the project and the colors and sounds I have chosen will be discussed in detail.

[4] For more see the appendix under "Historical Color-Sound Tables" of the PDF with updated and future versions on www.koulaouzidis.com/ARS-COMBINATORIA_K.Koulaouzidis.pdf.

4. Realization and Outlook

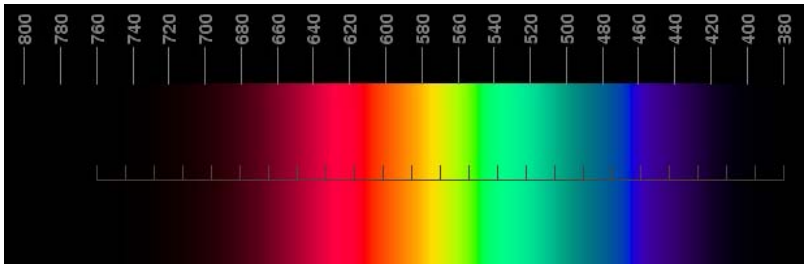
4.1 The Colors of the Project

Color can be mathematically defined by the electromagnetic wavelength to which it corresponds. Tests with a spectrometer bring the following results regarding the human perception of color:

	← UV (Ultraviolet)					(Infrared) IR →
Color	Violet	Blue	Green	Yellow	Orange	Red
Nanometer (nm)	390-450	450-492	492-577	577-597	597-622	622-780

As every human eye is different, and light conditions, reflections and other factors can alter the way one perceives color, the calculated, chosen and printed colors can only be an approximation.

For this project, I decided to use the color table below. It was generated by Dan Bruton's FORTRAN program (Bruton 2017). The program calculated approximate RGB values from the wavelengths between 380-780 nm of the light spectrum. I divided the table into pixels so that each pixel corresponds to a nanometer. Furthermore, I decided to use a part of the color spectrum said to be visible for man (about 390-760 nm) and also included the very faint visible parts of the ultraviolet and infrared portion, which are drifting towards black (because not really visible). This finally gives us the scope from 380-760 nm which I divided into 25 equal parts. At the end it was no longer a problem to extract the corresponding RGB values from this table.



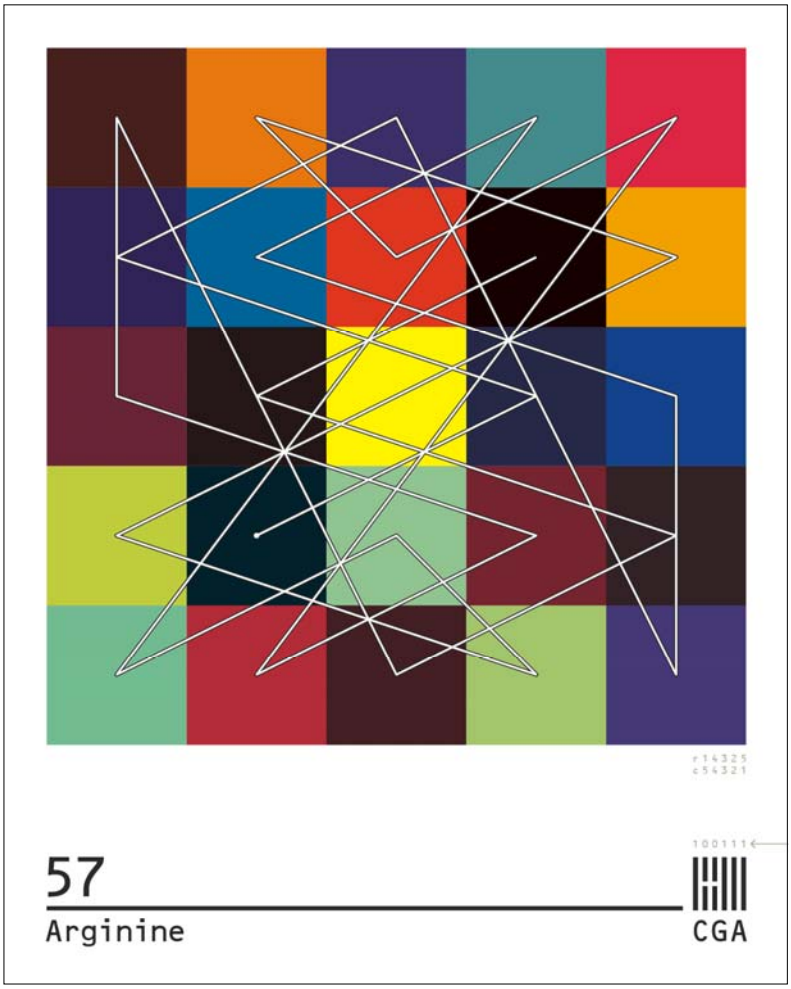
In order to produce colors as accurate as possible for the 64 print plates of the project (see next chapter) I used a calibrated screen and a spectrometer. There were quite a few test prints and adjustments of the printing profiles, but I finally achieved a very satisfying result!

For reference, this is the list/table of the 25 colors used in nm and their equivalent RGB values:

	R	G	B
01: 380,00 nm	0	0	0
02: 395,83 nm	0	0	5
03: 411,67 nm	10	0	25
04: 427,50 nm	30	0	70
05: 443,33 nm	55	0	120
06: 459,17 nm	55	0	190
07: 475,00 nm	0	85	155
08: 490,83 nm	0	125	135
09: 506,67 nm	0	180	140
10: 522,50 nm	0	230	150
11: 538,33 nm	0	255	125
12: 554,17 nm	110	255	0
13: 570,00 nm	230	235	0
14: 585,83 nm	255	155	0
15: 601,67 nm	255	75	0
16: 617,50 nm	255	0	45
17: 633,33 nm	235	0	65
18: 649,17 nm	170	0	50
19: 665,00 nm	110	0	35
20: 680,83 nm	65	0	15
21: 696,67 nm	35	0	5
22: 712,50 nm	20	0	0
23: 728,33 nm	5	0	0
24: 744,17 nm	0	0	0
25: 760,00 nm	0	0	0

4.2 Graphic Presentation: 64 Printed Alu-Dibond Boards

For the first time the results of long-term research and combination work were visualized and shown at the Goldberg Studios in Munich. They were exhibited from 23 to 26 July 2015 in the form of 64 graphic panels. This is one of the 64 exhibits.



The numbers of the squares contain a perfect harmony: they form absolutely symmetrical graphical patterns by linking the differently arranged numbers from 1-25 per square. This harmony can be transferred visually and logically to other areas as well. The colors from the uniformly subdivided light spectrum (in 25 parts) that are assigned to the numbers are subject to the displayed pattern and thus create a graphic unit: they too produce an optically perfect harmony. In addition, each panel shows the assignment of the numerical square to the binary system, I Ching, and base triplets with associated amino acids.

4.3 The Sounds of the Project

I needed 25 different sounds to assign them to the 25 colors, so after several experiments I decided to use five octaves of a pentatonic scale (from C2 to C6 in scientific pitch notation, in order for the pitch tone to be within the middle octave C4). According to the tempered mood these are defined as follows: the pitch tone (a1 / A4) is set to 432 hertz (Hz) and if the octave is divided mathematically into 5 distances of $2^{(1/5)} = 1{,}148698355$ the following 25 sounds can be used for the numbers of 1 to 25:

Table of Note Frequencies in Hertz (Hz)

71,25	81,85	94,02	108	124,06	→ C2
142,50	163,70	188,04	216	248,12	→ C3
285	327,40	376,08	432	496,24	→ C4
570	654,80	752,16	864	992,48	→ C5
1140	1309,60	1504,32	1728	1984,96	→ C6

4.4 Audiovisual Installation: 1394 Scores for Light and Sound

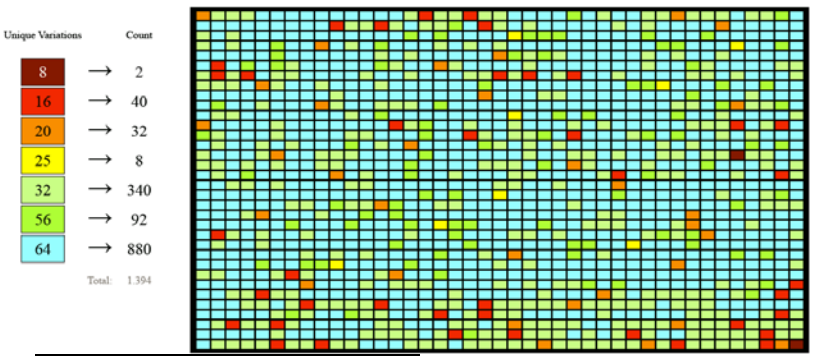
In the audiovisual installation[5] sounds are assigned to the colors from the graphical presentation of the project. If these are played according to certain rules, programmed music compositions (in the style of serial music) originate and harmoniously bring the colors to sound.

Each of the 64 squares (as described in C.) contains 1394 different combinations of 5 numbers from 1 to 25, which always have the sum 65 and are identical for each of the 64 squares. Each combination of numbers forms the basis of a single musical composition or score. Each of the 1394 scores consists of the same 64 sub-areas (analogous to the 64 squares) and plays five different tones per square (analogous to the five numbers per square with the sum 65) column by column. For each subarea of a score it is always the same 5 notes, but in different positions (depending on the position of the number in the square). This creates rhythm.

Similar to the twelve-tone technique, no tone is repeated until all five have been played, which is anyway the case for each of the 64 squares played. By means of this "five tone technique" each score has a playing time of 80 seconds. This results in a total running time of approx. 31 hours.

Calculations showed that there are certain groups within the 1394 scores, based on the different unique variations each square in a score plays back. Obviously, scores with less unique variations sound very repetitive and the ones with many unique variations sound more diverse.

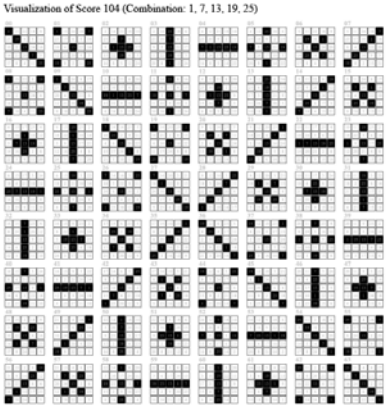
Color-coded list and overview (map) of all groups:

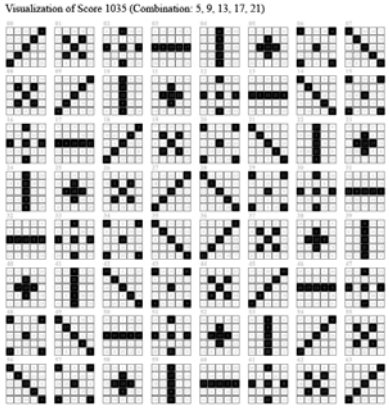


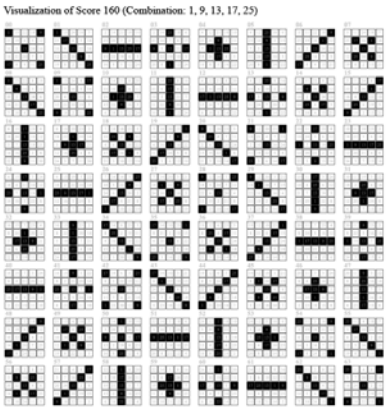
[5] The audiovisual installation will be exhibited at the Goldberg Gallery in Munich, 13-15.10.2017 and a presentation of the project will be held at the GGR-PY Conference, HfP (TUM), Munich, 14.10.2017.

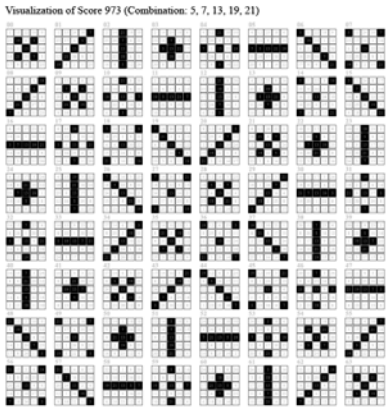
There is a symmetry even within a score group ("pair"-scores for all 1394 exist). Let us take, for example, the 8 scores of the group with the 25 unique variations (the yellow variations on the map) and how they look. Those are the scores with the numbers 104, 160, 319, 432, 759, 878, 973, 1035 and are paired as such: 104/1035, 160/973, 319/878, 432/759.

Each score is played from the left column to the right, sounding only the 5 corresponding notes of the score combination (marked in black). Where more than one note is on a column, those will be played simultaneously.

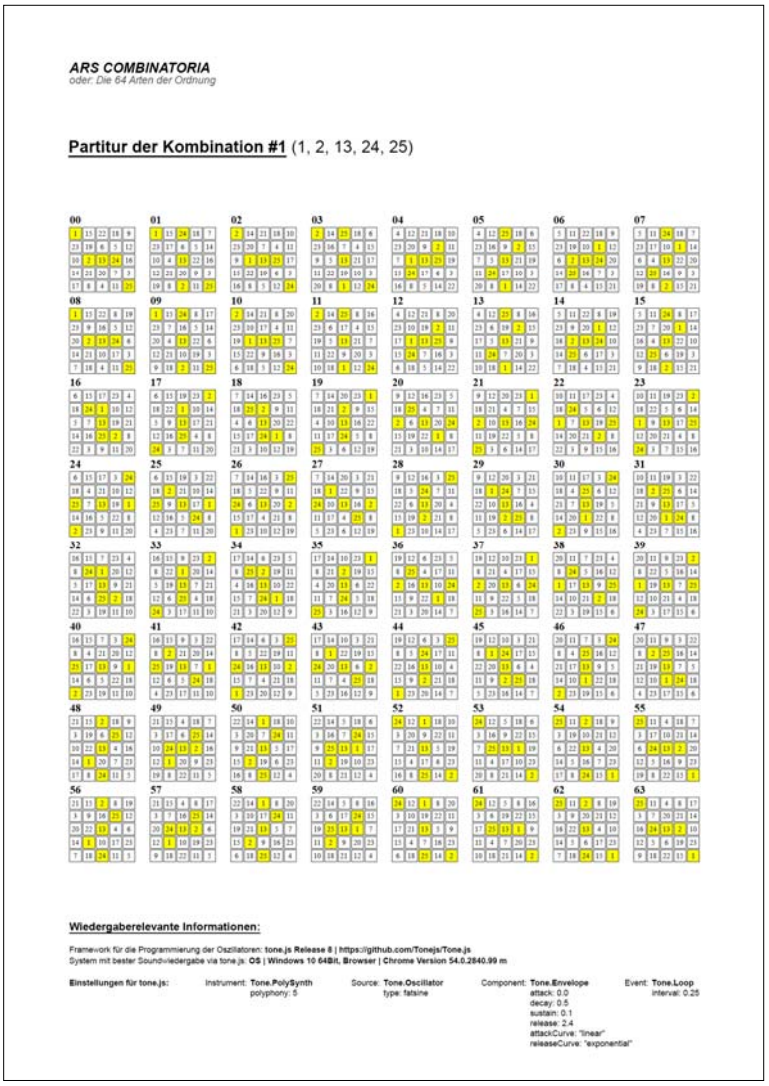








Example score sheet with settings for the instruments and oscillators (the sounding notes are marked in yellow).



ARS COMBINATORIA
oder: Die 64 Arten der Ordnung

Partitur der Kombination #1 (1, 2, 13, 24, 25)

Wiedergaberelevante Informationen:

Framework für die Programmierung der Oszillatoren: **tone.js Release 8 | https://github.com/Tonejs/Tone.js**
System mit bester Soundwiedergabe via tone.js: **OS | Windows 10 64Bit, Browser | Chrome Version 54.0.2840.99 m**

Einstellungen für tone.js:

Instrument: **Tone.PolySynth**
polyphony: 5

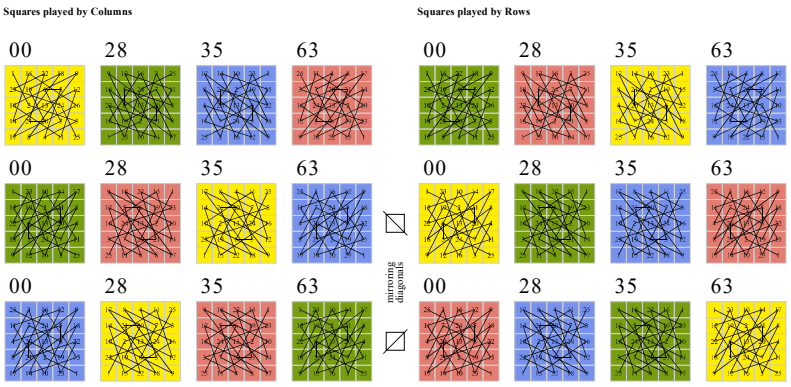
Source: **Tone.Oscillator**
type: fatsine

Component: **Tone.Envelope**
attack: 0.0
decay: 0.5
sustain: 0.1
release: 2.4
attackCurve: "linear"
releaseCurve: "exponential"

Event: **Tone.Loop**
interval: 0.25

Finally, one could ask why I was not playing the squares produced from mirroring across both diagonals as well. As I am showing in the following figure they do not produce any new musical scores, even if played back by rows instead of columns. Therefore, the initially chosen 64 squares without the 2 ways of mirroring are enough for my project.

As an example, we are using the first ultrasupermagic square "00" with its 3 rotational siblings "28", "35", "63", seen on the first row. On the second and third row you find the squares produced when the ones on the first row are mirrored across the two respective diagonals. Musical scores that sound identical have the same color.



5. Outlook

The next steps will be to concentrate on finalizing different audio-visual installations based on the many combinations of the 1394 scores.

Furthermore, I am very interested in collaboration with other artists as well as scientist and researchers interested in my work and the principles behind it. I am especially looking forward to collaborating with composers in the electronic and experimental music genre. In addition, I am still working on other types of orders like the application of laws in areas such as the linguistic alphabet, or an extended integration of the 64 base triplets of the essential amino acids of the genetic code. Further extensions will be presented on my website (www.koulaouzidis.com). Future exhibitions are planned. Stay tuned!

References (*Print Media)*

Barrow, J. D. (1998), *Impossibility: The Limits of Science and the Science of Limits*, Oxford: Oxford Univ. Press.
Conway, J.H. and R. Guy (1996), *The Book of Numbers*, New York: Springer Verlag.
Endres, F.C., and A. Schimmel, Annemarie (2001), *Das Mysterium der Zahl: Zahlensymbolik im Kulturvergleich*, München: Diederichs.
Ευαγγελόπουλος, Δημήτρης: Ιερή Γεωμετρία, Αθήνα: Αρχέτυπο / Μεταεκδοτική, 2002
Fadiman, C. (1958), *Fantasia Mathematica*, New York: Simon and Schuster,
Falkener, E. (1962), *Games Ancient and Oriental and how to play them*, New York: Dover Publications.
Gage, J. (2001), *Kulturgeschichte der Farbe – Von der Antike bis zur Gegenwart*, Leipzig: E.A. Seemann Verlag,
Gombrich, E. H. (1995), *The Story of Art,* London: Phaidon Press.
Goethe, J.W. von (2002), *Farbenlehre,* München: Hanser.
Gonzáles-Wippler, M. (1998), *The complete Book of Amulets & Talisman,* St.Paul: Llewellyn.
Haase, R. (1998), *Johannes Keplers Weltharmonik: der Mensch im Geflecht von Musik, Mathematik und Astronomie*, München: Diederichs.
Hahn, W. (1989), *Symmetrie als Entwicklungsprinzip in Natur und Kunst*, Königstein: Verlag Langewiesche.
Heinz, H. D., and J.R. Hendricks (2000), *Magic Square Lexicon: Illustrated*, Surrey: HDH.
Hesse, H. (1996), *Das Glasperlenspiel*, Frankfurt a.M.: Suhrkamp.
Hopper, V.F. (1969), *Medieval Number Symbolism*, New York: Cooper Square Publishers.
Hughes, R. (1991), The Shock of the New, London: Thames & Hudson.
Kayser, H. (1964), *Akroasis*, Basel: Schwabe.
Kandinsky, W. (1977), Concerning the Spiritual in Art, New York: Dover Publications.
Krausser, H. (1993), *Melodien*, München: Paul List Verlag.
Laßwitz, K(1998), *Die Universalbibliothek,* Hannover: Wehrhahn Verlag.
Maeda, J. (2000), *Maeda @ Media,* London: Thames & Hudson.
Maur, K. von (1999), *Vom Klang der Bilder*, München: Prestel.
McLeish, J. (1991), *Number*, London: Bloomsbury Publishing.
Meyer-Baer, K. (1984), *Music of the Spheres and the Dance of Death: Studies in Musical Iconology*, New York: Perseus Books.
Norman, R.B.(1990), *Electronic Color: The Art of Color Applied to Graphic Computing*, New York: Van Nostrand Reinhold Company.
Petit, M. (1997), *Uroboros*, Wien: Paul Zsolnay Verlag.

Plato: *Timäus.*
Ringborn, S. (1970), *The Sounding Cosmos: A Study in the Spiritualism of Kandinsky and the Genesis of Abstract Painting*. Åbo: Åbo Akademi.
Schönberger, M. (2000), *Weltformel I Ging und genetischer Code*, Aitrang: Windpferd.
Steiner, Rudolf (1976), *Das Wesen der Farben*, Dornach: Rudolf Steiner Verlag.
Stromer, K. (2002), *Farbsysteme*, Köln: DuMont.
Taylor, F.A .(1962): Colour Technology: For Artists, Craftsmen, and Industrial Designers, New York: Oxford University Press.
Taylor, T. (2003), Theoretic Arithmetic of the Pythagoreans, Whitefish: Kessinger Publishing.
Tocquet, R. (1961), *The Magic of Numbers*, New York: A.S. Barnes.
Werner, H. (2001), *Lexikon der Numerologie und Zahlenmystik*, Frechen: Komet.
Wilhelm, R. (2000), *I Ging - Das Buch der Wandlungen*, München: Hugendubel (Diederichs).
Woolman, M. (2000), *Sonic Graphics / Seeing Sound*, London: Thames & Hudson.

References (*Internet*)

Bruton, Dan: Color Science,
http://www.cox-internet.com/ast305/color.html
Bruton, Dan: RGB Values for Visible Wavelengths - FORTRAN Code,
http://www.physics.sfasu.edu/astro/color/spectra.html
Danielson, Holger: Magische Quadrate,
http://www.magic-squares.de/allgemeines/quadrate/anzahl.html
Goppold, Dr. Andreas: Music, Pattern, and the Neuro-Structures of Time,
http://www.uni-ulm.de/uni/intgruppen/memosys/symbol18.htm
Heinz, Harvey D.: Magic Squares, Magic Stars & Other Patterns,
http://www.geocities.com/~harveyh/
McClain, Ernest G.: Musical Theory and Ancient Cosmology,
http://www.new-universe.com/pythagoras/mcclain.html
Möllendorf, Willi: Musik mit Vierteltönen,
http://sonic-arts.org/monzo/moellendorf/book/theory.htm
Onion: Adaptive and Interactive Processes in Musical Systems,
http://www.antirom.com/onion
Suzuki, Mutsumi: Magic Squares,
http://mathforum.org/te/exchange/hosted/suzuki/MagicSquare.html