International Conference on

Nonlinear Phenomena in Biology, Ecology, Physics and Mechanics

in honour of Professor Messoud Efendiev
on the occasion of his 70th birthday

22-26 October 2023

Azerbaijan State University of Economics
(UNEC)

Sponsors:

Republic of Azerbaijan, Cabinet of Ministers

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Program

Sunday (October 22, 2023): Arrival of the participants

Monday (October 23, 2023)

Time	Activity
09:00 - 10:30	Visit of the tomb of national leader Heydar Aliyev and the alley of martyrs
10:30 - 11:00	Registration of the participants in the 3rd campus of UNEC Faculties
11:00 - 12:45	Opening Ceremony
12:45 - 14:00	Lunch in the 3rd Campus of UNEC Faculties
	Chair: Kumar Murty
14:00 - 14:20	Michael Cant: *War and Peace Since Life Began: How Intergroup Conflict Shapes the Evolution of Biological Societies*
14:25 - 14:45	Konstantin Khanin: *First Passage Percolation in Random Potentials of the Product Type*
14:50 - 15:10	Manfred Büchele: *Nonlinear Phenomena and Possible Significance in Fruit Growing Research*
15:15 - 15:35	Hermann Eberl: *A Spatio-Temporal Model of Fireblight During Bloom*
15:40 - 16:00	Coffee break
	Chair: Mads Peter Sørensen
16:00 - 16:20	Mitsuharu Otani: *Existence and Nonexistence of Global Solutions for Fujita-Type Equations with Some Nonlinear Boundary Conditions*
16:25 - 16:45	Massimo Grossi: *Critical Points of Semi-Stable Solutions of Laplace-Beltrami Operator on Surfaces*
16:50 - 17:10	Feridun Hamdullahpur: *Deep Convolutional Surrogates and Freedom in Thermal Design*
17:15 - 17:35	Hans Knüpfer : *Optimal size for magnetic domains in Thin Ferromagnetic Films*
17:40 - 18:40	Dinner in the 3rd Campus of UNEC Faculties
18:40 - 19:15	Drive to the Hajibeyov Baku Academy of Music
19:30 -	Concert at the Hajibeyov Baku Academy of Music

Tuesday (October 24, 2023)

Time	Activity
08:30	Bus transfer from hotel to UNEC campus
	Chair: Mitsuharu Otani
09:00 - 09:20	Sivabal Sivaloganathan: *A Coupled PDE Model of High Intensity Ultrasound Heating of Biological Tissue*
09:25 - 09:45	Sevda Isayeva: Transmission Acoustic Problems for Nonstationary Equations
09:50 - 10:10	Taylan Sengül: *Two Approaches to Instability Analysis of the Viscous Burgers' Equation*
10:15 - 10:35	Araz R. Aliev: *On the Conditions for the Existence of Smooth Solutions from Sobolev-Type Space for two Classes of Fourth Order Operator-Differential Equations*
10:40 - 11:00	Coffee break
	Chair: Carlos Rocha
11:00 - 11:20	Vahid Kadymov: *Contact Problems of Plastic Flow in a Thin Layer: Theory, Boundary Value Problems, Solutions and Applications*
11:25 - 11:45	Akiko Morimura: *Weak Solvability for Nonlinear Parabolic Equations Having Non-Monotone Boundary Conditions*
11:50 - 12:10	Tibor Krisztin: *Periodic and Connecting Orbits for MacKey-Glass Type Equations*
12:15 - 12:35	Yihong Du: *Propagation Dynamics of the Nonlocal KPP Equation with Free Boundaries*
12:40 - 14:00	Lunch in the 3rd Campus of UNEC Faculties
	Chair: Yihong Du
14:00 - 14:20	Carlos Rocha: *On the Structure of the Infinitesimal Generators of Semigroups with Discrete Lyapunov Functionals*
14:25 - 14:45	Varga Kalantarov: *Stablizaition of Solutions to Equations Modelling Dynamics of Viscoelatic Fluids*
14:50 - 15:10	Toyohiko Aiki: *New Mathematical Model for Elastic Collision*
15:15 - 15:35	Florian Rupp: *Modeling & Simulating the Stochastic Dynamics of Games with Three Pure Strategies*
15:40 - 16:00	Coffee break
	Chair: Sivabal Sivaloganathan
16:00 - 16:20	Radoslaw Czaja: *Finite-Dimensional Global and Exponential Attractors for Some Fourth Order Problems in* $\mathbb{R}^N$
16:25 - 16:45	Anibal Rodriguez-Bernal: *Homogeneous Spaces, Operators and Semigroups: Optimal Estimates, Spectral Analysis and Perturbations*
16:50 - 17:10	t.b.c.
17:15 - 17:35	t.b.c.
17:40 - 19:00	Walking tour in Icerisher (historic part of Baku)
19:00 - 22:00	Banquet at the Mugam Restaurant

Wednesday (October 25, 2023)

Time	Activity
08:30	Bus transfer from hotel to UNEC campus
	Chair: Hermann Eberl
09:00 - 09:20	Tomomi Yokota: *Boundedness in a Degenerate Chemotaxis System for Tumor Invasion*
09:25 - 09:45	Umayra Taghiyeva: *Climate Change as a Nonlinear Phenomenon*
09:50 - 10:10	Ali Ashher Zaidi: *Pantograph Type Partial Differential Equations and Tumor Growth*
10:15 - 10:35	Yoichi Enatsu: *Semi Wave Solutions for a Diffusive SI Model with Free Boundary*
10:40 - 11:00	Coffee break
	Chair: Florian Rupp
11:00 - 11:20	Stefanie Sonner: *Degenerate Reaction Diffusion Systems Modeling in Biofilm Growth*
11:25 - 11:45	Mads Peter Sørensen: *Reaction Diffusion Advection Induced Growth Patterns of Biofilms in a Centrifugal Lab-on-a-Disc Platform*
11:50 - 12:10	Ekkehard Holzbecher: *On Mass Transport in Sediments and Soil*
12:15 - 12:35	Elena Popova: *Stability of a Spacecraft Equipped with a Sail Moving Along an Intense Laser Beam*
12:40 - 14:00	Lunch in the 3rd Campus of UNEC Faculties

Thursday (October 26, 2023)

Time	Activity
08:30	Bus transfer from hotel to UNEC campus
	Chair: Toyohiko Aiki
09:00 - 09:20	Anzor Beridze: *On Axiomatic Homology Theory of General Topological Spaces*
09:25 - 09:45	Roland Duduchava: *Shell Equation Derived by the* Γ*-Convergence*
09:50 - 10:10	Kota Kumazaki: *On a Multiscale Problem Describing Water Swelling in Porous Materials*
10:15 - 10:35	Jacky Cresson: *Dynamics of Stochastic Hamiltonian Systems*
10:40 - 11:00	Coffee break
	Chair: Sivabal Sivaloganathan
11:00 - 11:20	Rövshan Khalilov: *Iron-Containing Nanoparticles in Biological Systems and its Biomedical Applications*
11:25 - 11:45	Aziz Eftehari: *Combination of in Vitro and Mathematical Investigations by Adverse Outcome Pathway to Identify Toxicants of the Mitochondria*
11:50 - 12:10	t.b.c.
12:15 - 12:35	t.b.c.
12:40 - 13:00	Closing ceremony

Abstracts

New Mathematical Model for Elastic Collision

Toyohiko Aiki

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Our aim of this talk is to discuss the elastic collision on a hard floor as an example of obstacle problems for elastic materials. For parabolic equations there are a lot of papers dealing with the obstacle problem. However, for the elastic materials most of results are concerned with the steady state, that is, time independent cases. Here, we note that in our previous works, we have introduced a mathematical model containing a stress function with singularity, and by numerical observations we showed that the model is available for large deformations. Also, for this model we establish some theoretical results on existence, uniqueness and large time behavior of the solutions. From these results we guess that by applying this kind of the stress functions, we can obtain the well-posedness of the obstacle problem. Thus, in this talk we propose a new boundary condition described by the singular stress function.

On the Conditions for the Existence of Smooth Solutions from Sobolev-Type Space for two Classes of Fourth Order Operator-Differential Equations

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In this paper, we study two classes of fourth-order operator-differential equations on the whole axis, the main part of which has multiple characteristics. The concepts of a smooth regular solution of order 1 and 'smoothly' regular solvability of these equations are introduced. In terms of the operator coefficients of the equations we establish sufficient conditions for their 'smoothly' regular solvability. At the same time, we find the exact

values of the norms of intermediate derivatives operators in a Sobolev-type space, which are closely related to the solvability conditions.

On Axiomatic Homology Theory of General Topological Spaces

Anzor Beridze

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On the category $\mathcal{K}^2_{CM}$ of pairs of compact metric spaces the exact homology theory was defined by N. Steenrod, that is known as the classical Steenrod homology theory. J. Milnor constructed the exact homology theory on the category $\mathcal{K}^2_{C}$ of pairs of compact Hausdorff spaces, which is isomorphic to the Steenrod homology theory on the subcategory $\mathcal{K}^2_{CM}$ and which satisfies the so-called "modified continuity" property: if $X_1 \leftarrow X_2 \leftarrow X_3 \leftarrow \dots$ is an inverse sequence of compact metric spaces with inverse limit X, then for each integer n there is an exact sequence:

$$0 \to \varprojlim{}^1 H_{n+1}(X_i) \xrightarrow{\beta} H_n(X) \xrightarrow{\gamma} \varprojlim H_n(X_i) \to 0,$$

where H_* is the Steenrod (Milnor) homology theory. There are exact homology theories defined by other authors (A. N. Kolmogoroff, G. Chogoshvili, K. A. Sitnikov, A. Borel and J. C. Moore, H. N. Inasaridze, D. A. Edwards and H. M. Hastings, W. S. Massey, E. G. Sklyarenko) that are isomorphic to the Steenrod homology theory on the category $\mathcal{K}^2_{CM}$ and so, satisfy the modified continuity axiom.

On the category $\mathcal{K}^2_{C}$ the axiomatic characterization is obtained by N. Berikashvili, L. Mdzinarishvili and Kh. Inasaridze, L. Mdzinarishvili, Kh. Inasaridze. The connection between these axiomatic systems is studied in the paper [1].

In the paper [2] we have generalized the result for general topological spaces. In particular, we have defined the Alexander-Spanier normal cohomology theory based on all normal coverings and have shown that it is isomorphic to the Alexandroff-Čech normal cohomology [2]. Using this fact and methods developed in [3], we constructed an exact, the so-called Alexander-Spanier normal homology theory $\bar{H}^N_*(-,-;G)$ on the category $\mathcal{K}^2_{Top}$, which is isomorphic to the Steenrod homology theory on the subcategory of compact pairs $\mathcal{K}^2_{C}$. Moreover, we gave an axiomatic characterization of the constructed homology theory [2]. In this paper we will use the method of construction of the strong homology theory to show that the homology theory $\bar{H}^N_*(-,-;G)$ is strong shape invariant.

The talk partially is based on joint works with co-authors Vladimer Baladze (BSU) and Leonard Mdzinarishvili (GTU).

References

[1] Beridze, Anzor; Mdzinarishvili, Leonard. On the axiomatic systems of Steenrod homology theory of compact spaces. *Topology Appl.* **249**, (2018), 73-82.

[2] Baladze, Vladimer; Beridze, Anzor; Mdzinarishvili, Leonard. On Axiomatic Characterization of Alexander-Spanier Normal Homology Theory of General Topological Spaces. Topology Appl. **317**, (2022), 1-25

[3] Beridze, Anzor; Mdzinarishvili, Leonard. On the Universal Coefficient Formula and Derivative $\varprojlim^{(i)}$ Functor. *arXiv:2102.00468*

Nonlinear Phenomena and Possible Significance in Fruit Growing Research

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Like many phenomena in nature, growth processes and natural live development cycles up to and including degeneration are not linear in fruit growing, but everything else. Cell divisions, as can be easily observed in bacteria, but also more complex growth of organisms and their organs often follow classical exponential functions, at least at the beginning. If several organisms are in competition for resources such as light, water, food or one organism parasitises another, then these functions overlap sometimes with the result a very steep hyperbola on the x-axis or an tochimg linear line. Natural influences from outside, such as temperature, humidity, as well as temporally different occurrences additionally modify the function or the expected result. The number of influencing factors is perhaps not quite 'legion' and not all oft hem are really important, but nature is very diverse.

In fruit growing research, people have been trying to make predictions for a long time. If epidemic developments, e.g. for the occurrence of animal, fungal and bacterial parasites in spring, could be continuously calculated according to weather conditions etc., then the optimal time for counteracting them could be determined. then it would be possible to determine the optimal time for countermeasures and whether the use of expensive pesticides is urgently required or even worthwhile. Considerable resources could be saved. In the fruit industry, we tend to work according to empirical values, with the damage threshold principle. Only when, for example, a certain number of winter eggs of a parasite are found on a plant plant protection against the pest should be carried out, if the weather permits as an additional factor. This is simple for the farmer, but not exactly sufficient for the growing mankind. And artificial intelligence needs more accurate data. We should therefore be able to predict all these processes in a continuously dynamised way for more complex parasites and their counterparts, especially abre for very dynamic diseases.

Another starting point for mathematical models is the optimal provision of resources for the growth of crops with the aim of maximum monetary yields, measured in quantity of products but also in quality, and all at low cost. The resources of water, fertiliser, artificial light and standing space are becoming scarce and have their price. In intensive systems such as soil-less cultivation or vertical farming, optimisation strategies are gaining in importance, but classical glasshouse systems or open agriculture in the ground must also be constantly optimised in terms of resource protection.

A third example is the post-harvest quality maintenance of apples or pears, for example. At least in temperate zones with seasons, there are cyclically occurring large harvest quantities from which the rest of the year is consumed. The shelf life, the quality of the fruit depends on various factors before, but also after harvesting. What proportion of the harvest can be kept for how long, how it develops in terms of quality under different conditions of temperature and gas mixture, and ultimately at what price the fruit is sold, can be decisive for the survival of a farmer or a fruit enterprise, but also of an entire fruit production region.

It will certainly be a challenge to transfer the theoretical findings and mathematically described processes into machines and applicable processes in the orchards. A particular challenge here is that the processes can hardly be isolated, as in a breeder reactor for medical agents. We usually work outside in nature. We need suitable sensors to determine the parameters and suitable tools that are controlled by an AI. This must be better than the intuition of a good gardener. In the best case, they support each other.

War and Peace Since Life Began: How Intergroup Conflict Shapes the Evolution of Biological Societies

Michael A. Cant

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The evolution of life on Earth is marked by a series of major transitions in which independent self-replicating units came together and started to reproduce as a collective: genes within genomes, cells within multicellular organisms, and individuals within societies. An outstanding goal of evolutionary biology is to explain how these transitions to new levels of collective organisation occur. In humans, theory and data suggest that an evolutionary history of lethal intergroup conflict – or warfare – might explain our own striking propensity to cooperate in large groups and with non-kin. Warfare is also a feature of social behaviour in many other social organisms, from bacteria to chimpanzees. Could warfare play a general role in social evolution, shaping societies to become more cohesive, cooperative, and closely integrated? Can nature provide insights into the factors that

promote peace? Here I describe my research on animal societies that aims to address these questions.

Dynamics of Stochastic Hamiltonian Systems

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We study the effect of stochastic perturbation in Hamiltonian systems using the notion of stochastic Hamiltonian systems. As an example, we study how the phenomenon of Arnold diffusion is modified in this context as well as Chirikov-Zaslavsky diffusion in skeleton Hamiltonian.

Finite-Dimensional Global and Exponential Attractors for Some Fourth Order Problems in $\mathbb{R}^N$

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A Cauchy problem for a dissipative fourth order parabolic equation in $\mathbb{R}^N$ is considered

$$u_t + \Delta^2 u = g(x) + m(x)u + f_0(x,u),\ t>0,\ x\in\mathbb{R}^N,$$

$$u(0,x) = u_0(x) \in H^2(\mathbb{R}^N),\ x\in\mathbb{R}^N,$$

with a mildly integrable potential function $m\colon \mathbb{R}^N \to \mathbb{R}$ such that

$$\|m\|_{L^r_U(\mathbb{R}^N)} = \sup_{y\in\mathbb{R}^N} \|m\|_{L^r(B(y,1))} < \infty$$

for some $\max\left\{\frac{N}{4},1\right\} < r \leq \infty$. Using the quasi-stability method by Chueshov and Lasiecka or the method from the joint paper with Messoud Efendiev [3] an estimate from above of the fractal dimension of a global attractor is derived. It is also shown that the global attractor is contained in a finite-dimensional exponential attractor.

Applying the same methods, we obtain similar results for the Cahn-Hilliard-Oono equation in $\mathbb{R}^N$:

$$u_t + \Delta(\Delta u + f(x,u)) + \delta u = 0,\ t>0,\ x\in\mathbb{R}^N,\ (\delta>0)$$

$$u(0,x) = u_0(x) \in H^1(\mathbb{R}^N),\ x \in \mathbb{R}^N,\ (N \leq 3)$$

for example for

$$f(x,u) = m(x)u - u|u|^{\rho-1},$$

where $\|m\|_{L^r_U(\mathbb{R}^N)} < \infty$ with $2 \leq r \leq \infty$ and

$$\rho > 1 \text{ arbitrarily large if } N = 1,2 \text{ and } 1 < \rho < \rho_c := \frac{N+2}{N-2} = 5 \text{ if } N = 3.$$

This is a joint work with Jan W. Cholewa based on the articles [1,2].

Suitably adapted considerations lead to corresponding results for a modified Swift-Hohenberg equation in $\mathbb{R}^N$ as was shown in cooperation with Maria Kania-Błaszczyk in papers [4,5].

References

[1] J.W. Cholewa, R. Czaja, On fractal dimension of global and exponential attractors for dissipative higher order parabolic problems in $\mathbb{R}^N$ with general potential, in *Contemporary Approaches and Methods in Fundamental Mathematics and Mechanics*, Understanding Complex Systems, Springer, 2021, 293–314.

[2] J.W. Cholewa, R. Czaja, Exponential attractor for the Cahn-Hilliard-Oono equation in $\mathbb{R}^N$, to appear in *Topol. Methods Nonlinear Anal.*

[3] R. Czaja, M. Efendiev, A note on attractors with finite fractal dimension, *Bull. London Math. Soc.* 40 (2008), 651–658.

[4] R. Czaja, M. Kania, Dissipative mechanism and global attractor for modified Swift-Hohenberg equation in $\mathbb{R}^N$, *Turkish J. Math.* 46 (2022), 2728–2750.

[5] R. Czaja, M. Kania, Exponential attractors for modified Swift-Hohenberg equation in $\mathbb{R}^N$, *Differential Integral Equations* 36 (2023), 347–366.

Propagation Dynamics of the Nonlocal KPP Equation with Free Boundaries

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The KPP equation has been used to model propagation since the pioneering works of Fisher and Kolmogorov-Petrovski-Piskunov (KPP). Significant further development of the theory has been done in the past decades along several directions. In this talk I'll report some recent work (with collaborators) on a nonlocal version of this equation equipped with suitable free boundaries, so that the population range of the propagating species is explicitly given in the model, with the propagating front represented by the free boundaries. A key new feature of this nonlocal model is that infinite-speed propagation may happen, and some sharp estimates of the propagation rate will be discussed in the talk.

Shell Equation Derived by the Γ-Convergence

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We consider boundary value problems (BVPs) of bending elastic isotropic thin media $\Omega^h : \mathcal{C} \times [-h, h]$ around a surface $\mathcal{C}$ with the Lipshitz boundary $\Gamma := \partial\mathcal{C}$, governed by the Láme equation

$$\begin{aligned} &\mathcal{L}_{\Omega^h}\mathbf{U}(x) = \mathbf{F}(x), && x \in \Omega^h := \mathcal{C} \times (-h, h),\\ &\mathbf{U}^+(t) = 0, && t \in \Gamma^h_L := \partial\mathcal{C} \times (-h, h),\\ &(\mathfrak{T}(t,\nabla)\mathbf{U})^+(t, \pm h) = \mathbf{H}(t, \pm h), && t \in \mathcal{C}. \end{aligned} \tag{1}$$

where $\mathfrak{T}(t,\nabla)$ is the traction operator and $\mathbf{U} = (U_1, U_2, U_3)^\top$ is the displacement.

The object of the investigation is what happens with the boundary value problems (1) when the thickness of the layer diminishes to zero $h \to 0$. Is proved the following.
THEOREM *Let the weak $\mathbb{L}_2$–limits*

$$\lim_{h\to 0} \mathbf{F}(t, h\tau) = \mathbf{F}(t), \qquad \lim_{h\to 0} \frac{1}{2h}\left[\mathbf{H}(t, +h) - \mathbf{H}(t, -h)\right] = \mathbf{H}^{(1)}(t) \tag{2}$$

exist. Then the boundary value problem (1) converges in the sense of Γ-limit to the following BVP on the mid surface $\mathcal{C}$

$$\begin{cases} \mu\left[\Delta_{\mathcal{C}}\overline{U}_\alpha + \mathcal{D}_\beta\mathcal{D}_\alpha\overline{U}_\beta - 2\mathcal{H}_{\mathcal{C}}\nu_\beta\mathcal{D}_\alpha\overline{U}_\beta - \mathcal{D}_\gamma(\nu_\alpha\nu_\beta\mathcal{D}_\gamma\overline{U}_\beta)\right] \\ +\dfrac{4\lambda\mu}{\lambda+2\mu}\left[\mathcal{D}_\alpha\mathcal{D}_\beta\overline{U}_\beta - 2\mathcal{H}_{\mathcal{C}}\nu_\alpha\mathcal{D}_\beta\overline{U}_\beta\right] = \dfrac{1}{2}F_\alpha + H^{(1)}_\alpha \quad \text{on} \quad \mathcal{C}, \\ \overline{U}_\alpha(t) = 0 \qquad\qquad \text{on} \quad \Gamma = \partial\mathcal{C}, \quad \alpha = 1, 2, 3. \end{cases} \tag{3}$$

where $\overline{\mathbf{U}}(t) := (\overline{U}_1(t), \overline{U}_2(t), \overline{U}_3(t))^\top$, $\overline{U}_\alpha(t) := U_\alpha(t, 0)$, $\alpha = 1, 2, 3$, λ, μ are the Láme constants, $\mathcal{D}_j := \partial_j - \nu_j\mathcal{D}_4$, $j = 1, 2, 3$ are the Günter's derivatives, $\mathcal{D}_4 := \partial_\nu = \sum_{k=1}^{3} \nu_k\partial_k$ is the normal derivative and $\Delta_{\mathcal{C}} = \mathcal{D}_1^2 + \mathcal{D}_2^2 + \mathcal{D}_3^2$ is the Laplace-Beltrami operator on the surface.

The operator in the ***shell equation*** (3) *is positive definite and the boundary value problem is uniquely solvable in the classical setting* $\overline{U} \in \mathbb{L}_2(\mathcal{C})$.

The suggested approach is based on the following: a) The variational representation of the BVP (1) (minimization problem for the energy functional); b) On the representation of the Laplace and Láme operators in terms of Günter's tangential and normal derivatives:

$$\Delta = \sum_{j=1}^{4} \mathcal{D}_j^2 + 2\mathcal{H}_{\mathcal{C}}\mathcal{D}_4, \qquad \mathcal{L} = -\mu\,\Delta_{\Omega^h} - (\lambda+\mu)\,\nabla_{\Omega^h}\mathrm{div}_{\Omega^h}.$$

Here $\mathcal{H}_{\mathcal{C}}$ is the mean curvature of the surface $\mathcal{C}$ and

$$\nabla_{\Omega^h}\varphi := \Big\{\mathcal{D}_1\varphi, ..., \mathcal{D}_4\varphi\Big\}^{\top}, \qquad \mathrm{div}_{\Omega^h}\mathbf{U} := \sum_{j=1}^{4}\mathcal{D}_j U_j, \quad U_4 := \langle \nu, \mathbf{U}\rangle$$

are the gradient and divergence.

The investigation is carried out in collaboration with T. Buchukuri (Tbilisi).

A Spatio-Temporal Model of Fireblight During Bloom

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Fireblight is a contagious disease that affects primary apples, but also pears and other members of the Rosaceae family.

Although the disease, caused by the gram negative bacterium *Erwinia amylovora*, has been documented and studied for more than 200 years, much is still not well understood about its spread in an orchard and how to control it. To this end we derive a spatio-temporal mathematical model for its propagation during bloom season, when pollinators are suspected to act as a mechanical vector of the pathogen. The model consists three of 3 ODEs in each point of the domain which describe the disease dynamics of the stationary host population. This is an SIR type model with nonlinear force of infection. It is coupled to two semilinear PDEs that describe the spread of the pathogen; one represents the bacteria on the surface of flower, the other one describes the pathogen entrapped in ooze that is produced by infected hosts. We present some numerical simulations of this PDE-ODE coupled model. These are strongly indicative of the possibility of Travelling Waves solutions. To confirm this mathematically, we construct upper and lower solutions as brackets around potential travelling waves, and use a fix-point argument.

This is joint work with Mike Pupulin, Xiang-Shen Wang, Messoud Efendiev and Thomas Giletti.

Combination of in Vitro and Mathematical Investigations by Adverse Outcome Pathway to Identify Toxicants of the Mitochondria

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Unfortunately, animal models frequently fail to detect mitochondrial toxicity, which emphasizes the necessity of predictive mathematical methods. In this work, we combine structure-based techniques with machine learning to present a model for predicting mitochondrial toxicity that focuses on the human mitochondrial respiratory complex. Knowledge about the progression of toxicity through layers of biological organization is arranged according to adverse outcome pathways (AOPs). AOPs lay the groundwork for mechanism-based alternative testing techniques for hazard assessment by identifying the connections between toxicity events at various levels. Here, we emphasize mitochondrial impairment to demonstrate the early stages of AOP development for consequences related to chronic toxicity.

Furthermore, the rise of big data in toxicology together with mitochondrial AOPs has been beneficial in creating in silico models that have improved interpretability and prediction performance. To be more precise, some methods combine in vitro data with structural information as model inputs, while others use structure-based models to predict the molecular initiating events of mitochondrial AOPs.

Improvements in machine learning techniques and the growth of in vitro databases containing pertinent biological knowledge have greatly improved in silico modeling for xenobiotic-induced mitochondrial toxicity prediction, even though mitochondrial dysfunction is still a challenging endpoint to predict.

Joint work together with Rövshan Khalilov.

Semi Wave Solutions for a Diffusive SI Model with Free Boundary

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Free boundary problems are used to model phenomena of biological invasion for species such as migration into a new habitat (e.g., Du and Lin (2010) and references therein).

Kim et al. (2013) proposed an SIR epidemic model with free boundary to investigate the front motion of infected individuals spreading into a region where disease is not prevalent. In this talk, we consider a diffusive epidemic model with a free boundary. We investigate the existence and nonexistence of a traveling wave solution of the model. We numerically observe the traveling wave and the front motion of the model.

This is a joint work with Takeo Ushijima and Emiko Ishiwata.

Critical Points of Semi-Stable Solutions of Laplace-Beltrami Operator on Surfaces

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We consider semi-stable *positive* solutions of the problem

$$\begin{cases} -\Delta_g u = f(u) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

Here M is a two-dimensional model space, which means $M = \mathbb{S}^2, \mathbb{R}^2$ or $\mathbb{H}^2$ with the corresponding standard metrics of constant curvature $1, 0, -1$, Δ_g is the Laplace-Beltrami operator and $\Omega \subset M$ is a *geodesically* convex set.

Under some suitable additional assumption we will show that the solution admits a *unique* critical point.

Joint work with Luigi Provenzano (Sapienza Universita' di Roma, Italy).

Deep Convolutional Surrogates and Freedom in Thermal Design

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A deep learning approach is presented for heat transfer and pressure drop prediction of complex fin geometries generated using composite Bezier curves. Thermal design process includes iterative high-fidelity simulation, which is complex, computationally expensive, and time-consuming. With the advancement in machine learning algorithms as well as Graphics Processing Units (GPUs), parallel processing architecture of GPUs can be used to accelerate thermo-fluid simulation. Convolutional Neural Networks (CNNs) are used to predict results of Computational Fluid Dynamics (CFD) directly from the saved topology images. A design space with a single fin as well as multiple morphable fins are studied. A comparison of Xception network and regular CNN is presented for the case with a single fin design. Results show that high accuracy in prediction is observed for single fin design particularly using Xception network. Xception network provides 98 percent accuracy in heat transfer and pressure drop prediction of the single fin design. Increasing the design freedom to multiple fins increases the error in prediction. This error, however, remains within three percent of the ground truth values which is valuable for design purpose. The presented predictive model can be used for innovative BREP-based fin design optimization in compact and high efficiency heat exchangers for involving complex geometries.

Mass Transport Processes in Sediments and Soil

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Many environmental problems, concerning contaminants, nutrients or biogeochemical species in general, are associated with mass transport in various environmental compartments that may be liquid, solid or gaseous. The transport equation that is used for the mathematical treatment describes, aside of storage, the physical processes of diffusion, resp. dispersion and advection. Source and sink-terms that may be linked to various biogeochemical processes, as well as nuclear decay can be added to the equation. In this contribution the focus lies on the compartments of soil and sediments, in which two phases, solid and fluid, are relevant for the behaviour of the compartments. It is outlined under which conditions the general description of the physical / biogeochemical system can be simplified to a single differential equation. The crucial part in the derivation is the isotherm, the relation between solid and fluid phase concentrations. The simple linear isotherm may leave the transport description linear, while the non-linear Langmuir

or Freundlich isotherms introduce (additional) nonlinear effects. It is outlined that for different soil and sediment compartments this nonlinearity affects advection, diffusion and reaction processes differently. Finally we illustrate some results of high interest for the solutions of the equations like self-sharpening fronts.

Transmission Acoustic Problems for Nonstationary Equations

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Mixed problems with transmission acoustic conditions for nonlinear wave equations are studied. The results on the existence and uniqueness of local weak and local strong solutions of the initial-boundary value problem with transmission acoustic transmission conditions for nonlinear wave equations with focusing sources were proved; results on the existence of global weak solutions and a result on the blow-up of weak solutions in a finite time for a mixed problem with transmission acoustic conditions for nonlinear wave equations with focusing sources were obtained; a result on the existence of a minimal global attractor for a mixed problem with transmission acoustic conditions for nonlinear wave equations with defocusing sources was obtained.

Contact Problems of Plastic Flow in a Thin Layer: Theory, Boundary Value Problems, Solutions and Applications

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A wide class of problems of plastic flow in a relatively thin layer between the approaching surfaces of tool bodies is considered. Such problems can be identified in a wide range of technological processes of material treatment by pressure, such as stamping and pressing of thin-walled structural elements, sheet rolling. These are complex nonlinear problems of plastic flow with a variety of parameters that determine the properties of the material in the layer. High pressures develop in the processes, so that in the initial approximation, the properties of the material are close to the properties of a hydrodynamic fluid. Practically, slippage is observed almost along the entire contact surface, at which the friction forces on the contact surface reach a maximum value equal to the yield strength of the layer material. The requirements for the accuracy of the final product determine another feature of these processes. As is known, large contact pressures cause normal elastic movements of the working surfaces of the tool bodies, commensurate with the thickness of the plastic layer. Consequently, the failure to take into account the elastic movements of the tool bodies becomes unjustified. In high-speed pressure treatment processes, viscosity and inertia forces play an essential role, which significantly complicate the modeling of the physical process. Temperature and heat flows play an important role in these processes. An effective, two-dimensional, averaged by the thickness of the current plastic layer theory is presented. A general boundary value problem, described by nonlinear partial differential equations of the second order, is formulated for a "viscous" fluid in a domain with a movable boundary.

It should be noted that until recently all research in this area was carried out on the basis of a simplified formulation of the boundary value problem in the model of an "ideal" fluid described by nonlinear partial differential equations of the first order, in which tangential stresses are neglected. Later, the simplified formulation was justified in the case of plastic flow over elastically deformable surfaces. The kinematics of the plastic layer spreading process is studied and a nonlinear evolutionary equation defining the free boundary of the spreading domain is derived. Other forms of representation of the evolutionary equation are presented; it is shown that this equation reduces to a special case of the nonlinear heat equation. New exact partial solutions of the evolutionary equation obtained using the methods of separation of variables and similarity transformations are written out. In conclusion, practical applications are given.

Stablizaition of Solutions to Equations Modelling Dynamics of Viscoelatic Fluids

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The talk is devoted to the problem of global exponential stabilization of solutions to the Navier-Stokes-Voigt equations

$$\begin{cases} v_t - \nu\Delta v - \alpha^2\Delta v_t + (v\cdot\nabla)v + \nabla p = -\mu w, \ x \in \Omega, t > 0, \\ \nabla \cdot v = 0, \quad x \in \Omega, t > 0 \end{cases} \tag{1}$$

and the second order in the Oscolokov system

$$\begin{cases} \partial_t(v - \alpha\Delta v) + (v\cdot\nabla)v - \nu\Delta u + \lambda\,(\partial_t^2 v + (v\cdot\nabla)\partial_t v) + \nabla p = -\mu w, \\ \nabla \cdot v = 0, \qquad x \in \Omega, t > 0 \end{cases} \tag{2}$$

where $\alpha > 0, \nu > 0, \mu > 0, m > 2, \lambda$ are positive parameters , $\Omega \subset \mathbb{R}^3$ is a bounded domain with sufficiently smooth boundary $\partial\Omega$ and w is a feedback control input (different for different problems).

We show that any arbitrary given solution of the initial boundary value problem for each of equations (1) and (2) can be stabilized by using feedback controllers depending only on finitely many large spatial-scale parameters and by controllers acting on a bounded subdomain $\omega \in \Omega$.

Iron-Containing Nanoparticles in Biological Systems and its Biomedical Applications

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As a result of biomineralization processes, nanoparticles of natural iron oxides are formed. These nanoparticles are widely distributed in nature and are present in many biological systems. The magnetic nanoparticles we are studying cause the formation of magnetic properties in living systems and lead to appearance emergence of electron paramagnetic resonance (EPR) signals, which were first discovered by us. B iogenic generation of nanophase magnetic oxides is connected with physiological processes including a number of pathologies, in particularly, human neurodegenerative diseases. In that connection, the investigation of mechanisms of formation iron oxide nanoparticles in living systems is important from fundamental viewpoint and also is necessary for development of novel efficient methods. Using EPR and TEM techniques we have demonstrated and studied the formation of ordered quasi-linear arrays of magnetic nanoparticles in immobilized DNA complexes via biomimetic reactions involving only biogenic compounds in aqueous solutions under normal conditions.

Also, the appearance of EPR signal characteristic for magnetic nanoparticles was found in various biosystems as a result of effects of a number of external stress factors.

Research of EPR signals of biosystems leaves showed that this method is promising method for detecting anomalous iron accumulation in plant systems in the form of magnetic nanoparticles and it provides new information for the diagnosis of various pathologies. The results of our studies play an important role in assessing the role of biogenic nanoparticles in plant systems.

Our results have important biomedical applications.

First Passage Percolation in Random Potentials of the Product Type

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In this talk we shall discuss asymptotic statistical properties of directed polymers in random environment of a product type. Namely, the random potential has the following form

$$\Phi(x,t) \;=\; F(x)B(t)\,, \qquad x,t \in \mathbb{Z}\,,$$

where $F(x)$ are iid random variables with compact support, and $B(t)$ are also independent for different t, and take values ± 1 with equal probability. It is also assumed that $\{F(x)\}$ and $\{B(t)\}$ are statistically independent.

From the physics point of view such models correspond to spatially disordered media which interact with the external time-dependent forces. The product structure corresponds to the situation when the spacial correlation length of the external forces is much larger than the correlation length of the spatially disordered media. Mathematical motivation is related to comparison with the KPZ case.

Directed polymers are important in studies of the random forced Burgers Equation and, closely related, the random Hamilton-Jacobi Equation and the stochastic Heat Equation. The case of first passage percolation corresponds to the inviscid setting when the viscosity ν vanishes.

We shall present several results describing limiting probability distributions for the polymer endpoint. These limit probability laws can be considered as analogues of the Tracy-Widom distribution in our model. We also prove that the shape function in our model exhibits phase transition from the linear to non-linear behaviour.

Joint work with Yuri Bakhtin, András Mészáros), and Jeremy Voltz

Optimal Size for Magnetic Domains in Thin Ferromagnetic Films

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We consider a regime of large and ultra-thin ferromagnetic films with strong anisotropy and easy axis pointing out of the film plane. Starting from the full three-dimensional micromagnetic energy, using the framwork of the calculus of variations and asymptotic analysis, we identify (to leading order) the critical scaling where the phase transition from single domain states to multi-domain states such as bubble or maze patterns occurs. The results of the analysis also include a rigorous proof for a scaling for the domain size. We also derive a reduced model in the framework of Gamma-convergence.

This is joint work with B. Brietzke, C. Muratov, F. Nolte and W. Shi.

Periodic and Connecting Orbits for Mackey–Glass Type Equations

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Delay differential equations of the form

$$y'(t) = -ax(t) + bh(y(t-1))$$

with positive real parameters a, b and a nonlinear function $h : [0, \infty) \to [0, \infty)$ arise in models for the dynamics of single species populations.

For monotone nonlinearities h the dynamics is relatively simple: There is a Morse decomposition of the global attractor, on each Morse component the dynamics is planar, and a fine structure of the global attractor can be described.

A nonmonotone, in particular a hump-shaped, or unimodal nonlinearity h can cause entirely different dynamics. An example is the hump-shaped $h(\xi) = \frac{\xi^k}{1+\xi^n}$, where k, n are positive parameters.

The case $k = 1$ was proposed in 1977 by Mackey and Glass as a model for the feedback control of blood cells. Since then this particular equation attracted the attention of many mathematicians interested in nonlinear dynamics and delay differential equations. The case $k = 2$ was introduced by Morozov, Banerjee and Petrovskii in 2016 as a minimal model to account for some of the most common features of population ecology whilst it remains capable of simulating sufficiently complex dynamics including long-living transients and chaos. Despite the large number of results showing convergence, oscillations, bifurcations, and complicated solution behavior, the dynamics is not understood yet. Most of the results are numerical.

We prove that for each $k \geq 1$ there are parameter values $b > a > 0$ such that, for sufficiently large n, orbitally asymptotically stable periodic orbits exist. The periodic orbits can be complicated in the sense that the projections $\mathbb{R} \ni t \mapsto (x(t), x(t-1)) \in \mathbb{R}^2$ can produce complicated looking figures.

In the case $k > 1$ an additional equilibrium point ξ^* arises, comparing to the case $k = 1$, from which there exist connecting orbits to zero and to the stable periodic orbit obtained in the first step. Heteroclinc connections are shown between periodic orbits. Moreover, we rigorously prove the existence of homoclinic orbits to ξ^*.

On a Multiscale Problem Describing Water Swelling in Porous Materials

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In this talk, we consider a multiscale problem describing water swelling in porous materials. This problem consists of a diffusion equation for the relative humidity distributed in materials and a free boundary problem describing the swelling process in microscopic pores. We consider each microscopic pore as a one-dimensional interval and correspond the interval to each point of materials. In our previous results, for given relative humidity we showed the well-posedness of the free boundary problem. In this talk, we impose a governing equation for the relative humidity and discuss the existence and uniqueness of a globally-in-time solution to this problem.

Weak Solvability for Nonlinear Parabolic Equations Having Non-Monotone Boundary Conditions

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In this talk we consider the initial and boundary value problem for the partial differential equations accompanying non-linear diffusion on the one-dimensional interval. Our research is strongly motivated from the mathematical model describing moisture transport in porous materials proposed by Green, Dabiri and Weinaug in 1970. They only reported validation of the model by comparing experimental and numerical results. Therefore, we aim to analyze this model, mathematically. Unknown functions of the model are distributions of water and air. For the model by assuming the air distribution, we obtain our present problem whose unknown function is only water distribution. However, we do

not expect existence of strong solutions to the problem since the boundary condition is not monotone. For this problem we define a solution by a weak formulation and prove existence and uniqueness. The existence is obtained by the standard fix-point argument and our proof of the uniqueness is based on the dual equation method.

This is a joint work with Toyohiko AIKI (Japan Women's University) and EBARA Corporation, Japan.

Existence and Nonexistence of Global Solutions for Fujita-Type Equations with Some Nonlinear Boundary Conditions

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In this talk, we are concerned with the existence and nonexistence of global solutions of the following Fujita-type equations with some special type of nonlinear boundary conditions.

$$\text{(P)} \quad \begin{cases} \partial_t u - \Delta u = |u|^{p-2}u, & t > 0,\ x \in \Omega, \\ -\partial_\nu u \in \beta^\alpha(u), & t > 0,\ x \in \partial\Omega, \\ u(0,x) = u_0(x) \geq 0, & x \in \Omega, \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$, $2 < p < \infty$, $\partial_\nu u = \nabla u \cdot \nu$ with ν the outward unit normal vector, and β^α is a family of maximal monotone graphs on $\mathbb{R}$ given by

$$\beta^\alpha(r) := \begin{cases} 0, & 0 \leq r < \alpha, \\ [0, +\infty), & r = \alpha, \\ \emptyset, & r \in (-\infty, 0) \cup (\alpha, +\infty). \end{cases}$$

The main purpose of this talk is to show the existence of a critical value $\alpha_c > 0$ such that

(1) $\alpha > \alpha_c \quad \Longrightarrow \quad$ there is no global solution for $\forall\, u_0 \in L^\infty_+$.

(2) $0 < \alpha < \alpha_c \quad \Longrightarrow \quad \exists\, u_0 \in L^\infty_+$ such that (P) admits a global solution.

Stability of a Spacecraft Equipped with a Sail Moving Along an Intense Laser Beam

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The Breakthrough Starshot Initiative proposed to develop the concept of propelling a nanoscale spacecraft using the radiation pressure of an intense laser beam. The nanocraft is a one-gram robotic spacecraft that consists of two main parts: the StarChip and the Lightsail. To achieve the goal of the project, it is necessary to solve non-trivial scientific problems. One of these tasks is to make sure that the nanocraft position and orientation inside the intense laser beam column are stable. The nanocraft driven by intense laser beam pressure acting on its Lightsail is sensitive to the torques and lateral forces reacting on the surface of the sail. These forces influence the orientation and lateral displacement of the spacecraft, thus affecting its dynamics. If unstable, the nanocraft might be expelled from the area of laser beam. In choosing the models for nanocraft stability studies, we are using several assumptions: (i) configuration of nanocraft is treated as a rigid body; (ii) flat or concave shape of circular sail; and (iii) mirror reflection of laser beam from surface of the Lightsail. We found conditions of position stability for spherical and conical shapes of the sail. The simplest stable configurations require the StarChip to be removed from the sail to make the distance to the center of mass of the nanocraft bigger than the curvature radius of the sail. Stability criteria do not require the spinning of the nanocraft. A flat sail is never stable.

On the Structure of the Infinitesimal Generators of Semigroups with Discrete Lyapunov Functionals

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Dynamical systems generated by scalar reaction-diffusion equations enjoy special properties that lead to a very simple structure for the semiflow. Among these properties, the monotone behavior of the number of zeros of the solutions plays an essential role. This discrete Lyapunov functional contains important information on the spectral behavior of the linearization and leads to a Morse-Smale description of the dynamical system.

Other systems, like the linear scalar delay differential equations under monotone feedback conditions, possess similar kinds of discrete Lyapunov functionals.

Here we discuss and characterize classes of linear equations that generate semiflows which preserve the order given by the discrete Lyapunov function.

This is based on a joint work with Giorgio Fusco (arXiv 2306.10403).

Homogeneous Spaces, Operators and Semigroups: Optimal Estimates, Spectral Analysis and Perturbations

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Homogeneous operators are those which interact with dilations in a certain way. Examples of such are the Laplacian, the bi-Laplacian and, more generally, the powers (including fractional powers) of the Laplacian in $\mathbb{R}^N$.

These operators and the semigroups they define, that is, the solutions of the associated parabolic equations, have some general remarkable properties that stem only from homogeneity.

Among them we will see that there are optimal estimates for the semigroups between homogenous spaces. Also we will show how homegeneity is reflected some important spectral properties, like accretivity or sectoriality. Finally, we will discuss homogenous perturbations of such operators and semigroups

This is a joint work with J. Cholewa (U. Silesia, Poland)

Modeling & Simulating the Stochastic Dynamics of Games with Three Pure Strategies

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Typically, randomness in evolutionary games is included by equipping the discrete deterministic replicator dynamics with Moran noise processes. In this talk, we extend this approach to obtain, from first principles, continuous evolution equations subject to white noise excitation. The modeling of these stochastic replicator dynamics will follow the approach applied in biological systems subject to system-internal random fluctuations. Our focus of study are evolutionary games with three pure strategies as they can be found by playing rock-scissor-paper or, in nature, by the territorial dynamics of the side-blotched lizards *Uta stansburiana*. Simulations of the stochastic game dynamics in its phase space together with discussions of the evolutionary stability of the equilibria will round-up the presentation.

The Spatio-Temporal Dynamics of Interacting Genetic Incompatibilities

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We explore the interaction between two genetic incompatibilities (underdominant loci in diploid organisms) in a population occupying a one-dimensional space. We derive a system of partial differential equations describing the dynamics of allele frequencies and linkage disequilibrium between the two loci, and use a quasi-linkage equilibrium approximation in order to reduce the number of variables.

We investigate the solutions of this system. We will show the interest of studying this system in the moving frames attached to the fronts to highlight phenomena such as front aggregation and acceleration induced by this aggregation.

Joint work with Matthieu Alfaro, Quentin Griette and Denis Roze.

Two Approaches to Instability Analysis of the Viscous Burgers' Equation

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The 1D Burger's equation with Dirichlet boundary conditions exhibits a first transition from the trivial steady state to a sinusoidal patterned steady state as the control parameter which controls the linear term exceeds 1. The main goal of this paper is to present two different approaches regarding the transition of this patterned steady state. As a first approach, we consider an external forcing on the equation which supports a sinusoidal solution as a stable steady state which loses its stability at a critical threshold. We use the method of continued fractions to rigorously analyze the associated linear problem. In particular, we find that the system exhibits a mixed type transition and steady state solutions which resemble a sinusoidal pattern are bifurcated. As a second approach, we consider the dynamics on the center-unstable manifold of the first two modes of the unforced system. In this approach, the secondary transition produces two branches of steady state solutions. On one of these branches there is another transition which indicates a symmetry breaking phenomena. This is a joint work with Burhan Tiryakioglu and Messoud Efendiev.

A Coupled PDE Model of High Intensity Ultrasound Heating of Biological Tissue

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Over the past decade, High Intensity Focused Ultrasound (HIFU) has emerged as an important novel therapeutic modality in the treatment of cancers, that avoids many of the associated negative side effects of more well-established cancer therapies (eg chemotherapy and radiotherapy). In this talk, a coupled system of partial differential equations is used to model the interaction of HIFU with biological tissue. The mathematical model takes into account the effects of both diffusive and convective transport on the temperature field, when acoustic (ultrasound) energy is deposited at a particular location (focal point) in the biological tissue. The model poses significant challenges in establishing existence and uniqueness of solutions. In this talk, we discuss well-posedness of our model, using the Leray-Schauder principle, together with a-priori estimates. We will also briefly discuss the long-time dynamics of solutions to this model, in the case where the external forcing is quasi-periodic. Here, we are able to prove the existence of uniform attractors to the corresponding evolutionary processes generated by our model and to estimate the Hausdorff?dimension of the attractors, in terms of the physical parameters of the system.

Degenerate Reaction Diffusion Systems Modeling Biofilm Growth

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Biofilms are dense aggregations of bacterial cells attached to a surface and held together by a self-produced matrix of extracellular polymeric substances. They affect many aspects of human life and play a crucial role in natural, medical and industrial settings. We

consider continuum models for spatially heterogeneous biofilm communities formulated as quasilinear reaction diffusion systems. Their characteristic feature is the two-fold degenerate diffusion coefficient for the biomass density comprising a polynomial degeneracy (as the porous medium equation) and a fast diffusion singularity as the biomass density approaches its maximum value. This degenerate equation is coupled to a semilinear reaction diffusion equation or an ordinary differential equation for the nutrient concentration. The latter case models cellulolytic biofilms where nutrients are immobilized in a cellulose surface that is degraded by the bacteria.

We present results on the well-posedness and regularity of solutions for such systems on bounded and unbounded domains. For systems with immobilized nutrients we also prove the existence of traveling wave solutions. Invading fronts had earlier been observed in biological experiments on cellulolytic biofilms as well as in numerical simulations of the model.

Joint work with J. Dockery (Montana State University, USA), H. Eberl (University of Guelph, Canada), J. Hughes (University of British Columbia, Canada), V. Hissink Muller and N. Lindemulder (Radboud University, Netherlands), and K. Mitra (Hasselt University, Belgium).

Reaction Diffusion Advection Induced Growth Patterns of Biofilms in a Centrifugal Lab-on-a-Disc Platform

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Experimental studies by Serioli et al have been performed on the growth pattern of bacteria (pseudomonas aeruginosa) in a rotating Lab-on-a-disc microfluidic platform. In the cell chamber the bacteria are present both in a free swimming planktonic state and embedded in growing biofilms [1,2]. It is observed experimentally that the biofilm growth happens predominantly on the walls of the perfusion chamber and in a spotted pattern.

Following the ideas in [3-5] we propose a mathematical model based on a reaction diffusion advection equation. In the model, the bacteria are divided into 3 states, which are a free swimming planktonic state, an active state and a dormant state. To this, we have added nutrition and antibiotics. The planktonic cells, nutrition and antibiotics are assumed to diffuse with constant diffusion coefficients, subject to advection. The active cells and the dormant cells are embedded in a biofilm and do not experience advection. Following the authors of references [3-5] we use a concentration dependent diffusion coefficient for the active and dormant cells in the biofilm. The diffusion coefficient is assumed singular or nearly singular.

Our numerical studies show spotted growth pattern in accord with the experimental observations by using the nonlinear and concentration dependent diffusion coefficient. This is in contrast to a linear and constant diffusion coefficient, which leads to a smooth and smeared growth pattern not in accord with experimental findings. Furthermore, the numerical simulations demonstrate growth predominantly at the walls of the perfusion chamber.

This is joint work together with Andreas Sølling Brendstrup, Jakob Tanderup, and Jonathan Aagren Jensen from the Department of Applied Mathematics and Computer Science of the Technical University of Denmark as well as Laura Serioli Øland, Kinga Zor, and og Anja Boisen from the Department of Health Technology of the Technical University of Denmark.

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Climate Change as a Nonlinear Phenomenon

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The speech discusses climate change as a nonlinear phenomenon affected by both natural and human factors. It highlights the global warming of 1.1°C above pre-industrial levels, primarily due to the burning of fossil fuels and irrational land and energy use practices over the past century.

The speech explains that the increase in global temperatures has led to various non-linear consequences, including more frequent and intense extreme weather events, which pose risks to nature and people. It emphasizes that these extreme events, such as heat-waves and heavy precipitation, become even more challenging when combined with other adverse events like pandemics or conflicts.

The speech notes recent unpredictable climate events, such as record heatwaves and floods, and emphasizes the need for improved early warning systems to mitigate their consequences. It highlights the positive impact of medical-meteorological warnings on reducing heat-related illnesses. The speech emphasizes the inadequacy of current efforts to combat climate change, given ongoing greenhouse gas emissions, and the focus on the losses and damages disproportionately affecting vulnerable populations and ecosystems.

The concept of climate-resilient development, integrating climate adaptation with emissions reduction, is introduced, with examples of its benefits, such as improved health and equality. The speech also mentions Azerbaijan's commitment to reducing emissions by 40% by 2050 and declaring liberated territories as net-zero emission zones.

In conclusion, the speech underscores the importance of future priorities and decisions in shaping our response to climate change, not only for the present but also for future generations.

Boundedness in a Degenerate Chemotaxis System for Tumor Invasion

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We consider a degenerate chemotaxis system for tumor invasion, which consists of four equations. The first equation has a nonlinear degenerate diffusion and density-dependent chemotactic sensitivity. We show that the system possesses a global bounded weak solution if the power of chemotactic sensitivity is weaker than the power of diffusion with difference of a constant depending the spatial dimension.

This is a joint work with Professor Sachiko Ishida (Chiba University).

Pantograph Type Partial Differential Equations and Tumor Growth

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We propose a mathematical model that reflects medical progress in tumor growth. The problem is motivated by a recent study that identifies two population cohorts in tumor tissue that play a key role in human tumor growth. The model captures the concept of the two-cohort interaction but is simple enough to study mathematically. The model involves nonlinear, nonlocal partial differential equations (PDEs). We develop tools to study properties of solutions to the problem. There is a paucity of mathematical information about such systems.

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