Thoughts on Voting Power and the Public Good Index

Matthias Weber

University of St. Gallen, matthias.weber@unisg.ch

Abstract: Among the wide variety of voting power indices, the public good index (PGI) is one of the less well-known ones. Holler (2019) posits hypotheses about why this is the case. In response to these hypotheses, I share a few thoughts about voting power in general and about the popularity of the PGI.

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1. Introduction

The Public Good Index (PGI; Holler 1978 and Holler 1982)[1] is one of multiple indices measuring voting power. Among these measures, the PGI is one of the less well-known and less used measures. Some people believe that this is unjustified and Holler (2019) posits hypotheses why the PGI may be less appreciated than deserved.

Some of these hypotheses relate to technical properties of the PGI, some others concern the importance of who introduced the index and how it was it classified. Manfred Holler asked for comments on his hypotheses. In this short comment, I will first share some rather general thoughts about voting power and the PGI that relate to part of Manfred Holler's

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[1] For a discussion of the public good nature, see Holler and Packel (1983).

hypotheses and afterward comment directly on the hypothesis that the PGI is underappreciated because it was not introduced by a famous game theorist.

2. Important Properties of Voting Power Indices

Some of Manfred Holler's hypotheses about why the PGI has received relatively little attention are concerned with the technical properties of the index itself and with the value that has been placed on some characteristics of power indices rather than on others (e.g., on local monotonicity). Rather than discussing these properties here, I will discuss another property that power indices can possess, which is in my opinion of crucial importance. I will then later get back to applications of power indices and the PGI. The property that I consider of utmost importance in a power index is to coincide with some form of outcomes. Most power indices do not have this property – at least, when the motivations usually brought forward for these indices are taken at face value, power differences between groups or individuals are different from outcome differences (outcomes could be some sort of utility, in the simplest case linear transferable utility).

Much of the voting power literature is normative in nature. That is, claims are made about which voting systems are fair based on their difference in voting power. However, as these differences in voting power are not differences in outcomes, this seems plainly wrong. Looking at the standard motivation of Penrose's Square Root Rule for two-tier voting systems (Penrose 1946; Banzhaf 1965; for concise descriptions see Turnovec 2009 or Weber 2016), this becomes particularly clear. If you are an individual who favors the adoption of a proposal and the proposal is not adopted, how do you benefit from the representative of your group having been responsible for the failure to adopt the proposal? Not at all, of course. Probably some scholars noticed already a long time ago that the probability of indirectly influencing the outcome in such a setting is different from the probability of getting what one wants, but many certainly did not. Therefore, voting power differences may have looked like an acceptable normative criterion for judging the fairness of voting systems. However, since Laruelle and Valenciano (2005), who refer to the distinction between power and outcomes as decisiveness versus success, or Barbera and Jackson (2006) one can no longer pretend that minimizing

voting power differentials is a useful normative criterion.[2] In short, after these works, one cannot continue as before! Unfortunately, some scholars seem not to be fully aware of this. Sometimes, one can even see footnotes of the sort "we know that ex-ante power is not identical to ex-post power but follow the literature in sticking with ex-ante power as normative concept." This is wrong. All valid normative concepts in the voting (power) literature now must be outcome-based. Except for the Shapley-Shubik index (Shapley and Shubik 1954) with its motivation in so-called divide-the-dollar games, voting power indices and their motivations are usually not outcome-based. I believe that this is a strong and valid reason for the popularity of the Shapley-Shubik index.

This does not necessarily mean that there is no more scope for classical voting power indices. Even indices such as the Banzhaf index or the PGI that do not possess an outcome-based motivation may prove useful, if they are not used as normative fairness criteria but for positive analyses. Of course, in a rational world, which economists have almost exclusively assumed for multiple decades, the normative and positive criteria should coincide. However, there is by now a vast literature documenting boundedly rational behavior in economic and political situations (accumulated since at least Tversky and Kahneman 1974 and Grether and Plott 1979). Even more, it seems that psychological factors are of crucial importance in *all* economic fields.[3] As focusing on actual human behavior, in contrast to assuming rational choice, has proved so valuable in all these fields, there is no reason why actual human behavior should not be relevant in the field of voting power. This means that voting power indices can still be very useful if they have a meaningful positive side to them!

In a way, Weber (2020) can be interpreted as an attempt to "save" the Banzhaf index with a positive analysis. In this work, I investigate

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[2] This is hinted at diplomatically in Kurz et al. (2015): "Social choice articles now appearing in top economics journals are concerned first and foremost with the welfare properties of voting systems; power comes as a distant second or even third ... But welfarist approaches to voting, which focus on measures of success rather than pivotality, can be viewed as part of power index research defined in a sufficiently expansive way."

[3] This includes public finance (Weber and Schram 2017), asset pricing (Weber et al. 2018, Kopányi-Peuker and Weber, *forthcoming*), and macroeconomics (Hommes et al. 2019; Bertasiute et al. 2020). Even when the behavior of scholars themselves is concerned, where one should maybe expect the most rational behavior, non-rational perceptions are important (Weber 2018).

preferences over voting systems for assemblies of representatives. More precisely, I investigate whether participants in an incentivized laboratory experiment prefer voting systems approximating equal indirect Banzhaf power over voting systems approximating equal indirect Shapley-Shubik power in a setting that is as close as possible to the standard motivation of the Banzhaf index (or Penrose's Square Root Rule). My thought was that, while the Banzhaf index is (from a theoretical viewpoint) worthless as a normative criterion, it may still describe well what kind of voting systems people prefer. If many political scientists, mathematicians, and economists found this concept convincing for multiple decades (as normative criterion), it may be possible that this concept still coincides with people's feeling of fairness. Unfortunately for the Banzhaf index, participants in Weber (2020) prefer voting systems approximating equal indirect Shapley-Shubik power over those approximating equal indirect Banzhaf power. Of course, this is only a comparison between these two concepts. It could well be that voting systems designed according to other concepts, that which may include those based on the PGI, are preferred as much as or even more than voting systems approximating equal indirect Shapley-Shubik power.

Analyzing people's preferences over voting systems is not the only possible positive analysis of voting. Other options include analyzing whether power indices have predictive power concerning which coalitions form in democratic institutions (although it may prove difficult to disentangle the contributions of political proximity and voting power to the forming of coalitions). In reality, it is well possible that smaller parties have greater actual power to influence politics than larger parties (which is in theory possible with the PGI but not with the Banzhaf or Shapley-Shubik index). Looking for example at Germany, one could easily make the claim that the libertarian party FDP has had more power in the past than other parties despite often rather small vote shares, as the FDP has often been part of the governing coalition (as junior partner of the large conservative party CDU).[4] The intuition of the PGI in mind, this is not necessarily surprising. The small vote share of the FDP was enough to give a coalition of CDU and FDP the majority of votes in parliament, so

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[4] For simplicity, I refer to the CDU only, although more precise would be to talk about the union of CDU and CSU. The CSU is officially a separate party that can be elected in Bavaria only (while the CDU can be elected in all states except Bavaria).

that it would have been unnecessary for the CDU to coalesce with a larger party than the FDP or with more than one party. Indeed, when the PGI was first brought up (Holler 1978), the task was exactly to analyze government formation through voting power indices (for Finland 1948-1978).

However, as mentioned above, it is difficult in such real-world applications to disentangle pure theoretical power considerations from political proximity. In the German example, one should not forget that the FDP is also the party that is politically closest to the CDU. It could thus be coincidental that the FDP had fewer votes than parties ideologically farther from the CDU and that political proximity determined the forming of the CDU-FDP coalitions. The analysis in Holler (1978) focuses on theoretical power alone, without taking political proximity into account.[5] To obtain a powerful predictor of coalition formation, it seems necessary to combine power considerations with measures of political proximity. Nowadays, such measures may be available or can be calculated. This makes it possible to analyze power indices and political proximity jointly, for example in regression settings. It is not a trivial task to use voting power theory for forecasting coalition formation, but that does not mean that it is not a worthwhile task. I believe that some voting power indices that are not valid normative criteria could prove useful for such tasks, including the PGI.

3. Reinhard Selten's View on the Missing Popularity of the PGI

The first hypothesis mentioned in Holler (2019) originates from Reinhard Selten and states that the PGI would be used and appreciated much more, if it had been introduced (and advertised, I would add) by a famous game theorist (from the US, I would add here). This case seems clear-cut. Unfortunately, in the social sciences (at least in economics and finance, the fields that I am most familiar with, but probably also in other social sciences), there is a very strong focus on scholars from top schools and on scholars from the United States. I have little doubt that there is a

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[5] Voting power considerations may be more important than political considerations in some cases. One example could be the recent coalition of Italy's far right with the far left. However, one may also claim that these parties have a lot in common, reaching from their populistic rhetoric over their contempt for democratic values and institutions, so that they may not be as distant as a simple left-right distinction

considerable difference in popularity of two equally good concepts if one is introduced by a relatively unknown European author and the other by a famous US American author. I hope that this will change at some point, that ideas from less known scholars will be taken more seriously, and that the region of origin of a concept will not matter, but I am a bit pessimistic, at least regarding the short and medium run. As far as the regional component is concerned, cold comfort for Europeans could be that Asians, South-Americans, and Africans are most likely in an even worse position.

I therefore agree with Reinhard Selten's view that the PGI would be more popular had it been introduced by a leading game theorist in the US. Whether this effect is so strong that the PGI would have become more popular than the Banzhaf index or even as popular as the Shapley-Shubik index is a different question.

4. Concluding Remarks

Should the PGI be used more often? Maybe yes. Rather than discussing the technical properties of this index, I have tried to make another point in this short comment: voting power indices should be used for positive analyses, as almost all of them are not valid normative criteria. They can be useful, though, in describing people's preferences over voting systems (as in Weber 2020) or for forecasting which coalitions will form. The latter was the purpose of the work introducing the PGI (Holler 1978), but successful modern applications may want to combine the voting power considerations with considerations of political proximity.

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